MA8151 – MATHEMATICS – I

QUESTION BANK

UNIT I DIFFERENTIAL CALCULUS

1. Sketch the graph and find the domain and range of each function: **(a) $f(x) = 2x - 1$ (b) $f(x)= x^2$**

2. Find the domain of each function: (a) $f(x) = \sqrt{x+2}$ (b) $g(x) = \frac{1}{x^2 - x}$

3.Evaluate the limit if it exists

(a) $\lim_{h\to 0} \frac{(-5+h)^2 - 25}{h}$ (b) $\lim_{x\to 16} \frac{4-\sqrt{x}}{16x - x^2}$ (c) $\lim_{x\to -4} \frac{\sqrt{(x^2+9)} - 5}{x+4}$

4. Use the Squeeze theorem to show that $\lim_{x\to 0}[x^2 \cos 20\pi x] = 0$

5. Where are the following functions continuous? a) $h(x) = \sin(x^2)$ b) $F(x) = \ln(1+\cos x)$

6.Find the equation of the tangent line to the hyperbola $y = \frac{3}{x}$ at the point (3, 1).

7. Find the equation of the tangent line to the parabola $y = x^2 - 8x + 9$ at (3, -6).

8. If $f(x) = x^3 - x$, find a formula for $f'(x)$

9. a) If $f(x) = \sqrt{x}$, find the derivative of f. State the domain of f'

(b) Find f' if $f(x) = \frac{1-x}{2+x}$

10. Where is the function $f(x) = |x|$ differentiable?

11. If $f(x) = x^3 - x$, find and interpret $f''(x)$.

12. Find the equations of the tangent line and normal line to the curve $y = x\sqrt{x}$ at the point (1, 1). Illustrate by graphing the curve and the lines.

13. Find the points on the curve $y = x^4 - 6x^2 + 4$ where the tangent line is horizontal.

14. Find a cubic function $y = ax^3 + bx^2 + cx + d$ whose graph has horizontal tangents at the points (-2, 6) and (2, 0).

15. At what point on the curve $y = e^x$ is the tangent line parallel to the line $y = 2x$?

16. Find an equation of the normal line to the parabola $y = x^2 - 5x + 4$ that is parallel to the line $x - 3y = 5$.

17. Find the equations of both lines that are tangent to curve $y = 1 + x^3$ and is parallel to the line 12x-y = 1.

18. For what values of x is the function $f(x) = |x^2 - 9|$ differentiable ? Find formula for f'

19. *Where is the function f(x=) differentiable? Give a formula for* f'.

$f(x) = |x-1| + |x+|$

20. A tangent line drawn to the hyperbola xy = c at a point P.

(a) Show that the midpoint of the line segment cut from this tangent line by the coordinate axes is P.

(b) Show that the triangle formed by the tangent line and the coordinate axes always has the same area, no matter where P is located on the hyperbola.

21. a. If $f(x) = xe^x$, find $f'(x)$

b. Find the nth derivative $f^{(n)}(x)$

c. If $f(x) = \sqrt{x}\, g(x)$, where $g(4) = 2,\ g'(4) = 2$ find $f'(4)$.

22. a. Find y' if $y = \dfrac{x^2 + x - 2}{x^3 + 6}$

b. Find an equation of the tangent line to the curve $y = \dfrac{e^x}{1+x^2}$ at the point $\left(1, \dfrac{e}{2}\right)$.

23. Find $f'(x)$ if $f(x) = \tan x$, $f(x) = \cot x$, $f(x) = \sec x$ ...

24. Calculate 1. $\lim\limits_{x\to 0} \dfrac{\sin 7x}{4x}$ 2. $\lim\limits_{x\to 0} x \cot x$ 3. $\lim\limits_{x\to 1} \dfrac{\sin(x-1)}{x^2 + x - 2}$

4. $\lim\limits_{x\to \frac{\pi}{4}} \dfrac{1 - \tan x}{\sin x - \cos x}$ 5. $\lim\limits_{x\to 0} \dfrac{\sin x^2}{x}$ 6. $\lim\limits_{x\to\infty} x \sin\left(\dfrac{1}{x}\right)$

25. Find $F'(x)$ if

(1) $F(x) = \sqrt{x^2+1}$ (2) $F(x) = \sin(x^2)$ (3) $F(x) = \sin^2 x$ (4) $F(x) = (x^3 - 1)^{100}$

(5) $F(x) = \dfrac{1}{\sqrt[3]{x^2 + x + 1}}$ (6) $F(x) = (2x+1)^5 (x^3 - x + 1)^4$ (7) $F(x) = e^{\sin x}$

27.Find the equation of the tangent to the circle $x^2+y^2=25$ at (3, 4)

28.Find y' if $x^3+y^3=6xy$

(8) $F(x)=\sin(\cos(\tan x))$

26. If $x^2+y^2=25$, find $\frac{dy}{dx}$

29.Find the tangent to the folium of Descartes $x^3+y^3=6xy$ at the point (3, 3). Also find at what point in the first quadrant is the tangent line horizontal.

30.Find y' if $\sin(x+y)=y^2\cos x$

31.Find y'' if $x^4+y^4=16$

32. Find where the function $f(x)=3x^4-4x^3-12x^2+5$ is increasing and where it is decreasing.

33. Find the local maximum and minimum values of the function $g(x)=x+2\sin x,\ 0\le x\le 2\pi$

34. Discuss the curve y = x^4-4x^3 with respect to concavity, points of inflection and local maxima and minima. Use the information to sketch the curve.

35. Sketch the graph of the function $f(x)=x^{2/3}(6-x)^{1/3}$

UNIT-II FUNCTIONS OF SEVERAL VARIABLES

PART A

(1) Find $\frac{dy}{dx}$ if $x^y + y^x = 1$

(2) If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$

(3) If $w = f\left(y-z, z-x, x-y\right)$ then show that $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$

(4) Given the transformation $u = e^x \cos y$ & $v = e^x \sin y$ and that f is a function of u and v and also of x and y, prove that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \left(u^2 + v^2\right)\left(\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2}\right)$

(5) If z is a function of x and y where $x = e^u + e^{-v}$ & $y = e^{-u} - e^v$ prove that

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - z\frac{\partial z}{\partial y}$$

(6) If $u = (x-y)f\left(\frac{y}{x}\right)$ then find $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2}$

(7) 1. Expand $e^x \cos y$ in powers of x and y as far as the terms of third degree.

PART B

1. Find the Taylor's series expansion of x^y near the point (1, 1) up to the second degree terms.
2. Find the Taylor's series expansion of $e^x \sin y$ near the point (-1, $\frac{-}{4}$) up to the third degree terms.
3. Find the Taylor's series expansion of $x^2y^2 + 2x^2y + 3xy^2$ in powers of $(x+2)$ and $(y-1)$ up to the third powers.
4. Using Taylor's series, verify that $\log(1+x+y) = (x+y) - \frac{1}{2}(x+y)^2 + \frac{1}{3}(x+y)^3 - \ldots$
5. Expand $e^x \log(1+y)$ in powers of x and y up to the third degree terms using Taylor's theorem.
6. Expand $e^x \cos y$ at $(0, \frac{\pi}{2})$ up to the third term using Taylor's series.

7. Obtain the Taylor's series of $x^3 + y^3 + xy^2$ in powers of $x-1$ and $y-2$.

8. Expand $\sin(xy)$ at $(1, \frac{\pi}{2})$ up to second degree terms using Taylor's series.

9. Expand $e^x \sin y$ in powers of x and y up to the third degree terms.

10. Expand $\sin(xy)$ in powers of $x-1$ and $y-\frac{\pi}{2}$ up to second degree term by Taylor's theorem

(8) If $x = r\cos\theta$ and $y = r\sin\theta$, verify that $\frac{\partial(x,y)}{\partial(r,\theta)} \cdot \frac{\partial(r,\theta)}{\partial(x,y)} = 1$

9. If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$ and $z = r\cos\theta$, find $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$

10 If $u = 2xy$, $v = x^2 - y^2$, $x = r\cos\theta$ and $y = r\sin\theta$, compute $\frac{\partial(u,v)}{\partial(r,\theta)}$.

11 Find the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3 if $y_1 = \frac{x_2 x_3}{x_1}$, $y_2 = \frac{x_3 x_1}{x_2}$, $y_3 = \frac{x_1 x_2}{x_3}$

12. If $x+y+z=u$, $y+z=uv$ and $z=uvw$,find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$

13. Discuss the maxima and minima of $f(x,y) = x^3y^2(12-x-y)$.

14. Find the extreme value of the function $f(x,y) = x^3 + y^3 - 3x - 12y + 20$.

15. Examine $f(x,y) = x^3 + 3xy^2 - 12x^2 - 15y^2 + 72x$ for extreme values.

16. Investigate the extreme values of the function $f(x,y) = x^2 + xy + y^2 + \frac{1}{x} + \frac{1}{y}$.

17. Discuss the extreme values of the function $f(x,y) = x^2 - 2xy + y^2 + x^3 - y^3 + x^4$ at the origin.

18. Identify the saddle point and the extreme points of $f(x,y) = x^4 - y^4 - 2x^2 + 2y^2$

19. A rectangular box, open at the top, is to have a volume $32cc$. Find the dimensions of the box, that require the least material for its construction.

20. Using Lagrange's multipler method, determine the maximum capacity of a rectangular tank, open at the top, if the surface area is $108m^2$.

21. Find the maximum value of $x^m y^n z^p$ when $\varphi = x+y+z-a$.

22. Find the maximum value of x^2yz^3 subject to $2x+y+3z=a$.

23.Find the maximum and minimum of $x^2 + y^2 + z^2$ subject to the condition $ax + by + cz = p$.

24. Find the shortest and the longest distances from the point $(1, 2, -1)$ to the sphere $x^2 + y^2 + z^2 = 24$.

25. Find the volume of the greatest rectangular parallelepiped inscribed in the ellipsoid whose equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

26. Find the length of the shortest line from the point $(0, 0, 9)$ to the surface $z = xy$.

27. Find the extreme value of $x^2 + y^2 + z^2$ subject to the condition $x + y + z = 3a$.

28. The temperature T at any point (x, y, z) in space is $T = cxyz^2$, where c is a constant. Find the highest temperature on the surface of the sphere $x^2 + y^2 + z^2 = 1$.

UNIT III INTEGRAL CALCULUS

PART A

1. For the region S bounded by $y = x^2$, x = 0, x = 1 and x-axis, show that the sum of the areas of the upper approximating rectangles approaches to $\frac{1}{3}$. i. e. $\lim_{n\to\infty} R_n = \frac{1}{3}$.

2. Let A be the area of the region that lies under the graph of $f(x) = e^{-x}$ between x = 0andx= 2.
   a) Using right end points, find an expression for A as a limit.
   b) Estimate the area by taking the sample points to be mid points and using four subintervals and then ten subintervals

3. Express $\lim_{n\to\infty} \sum_{i=1}^{n} (x_i^3 + x_i \sin x_i)\Delta x$ as an integral on the interval $[0, \pi]$.

4. Evaluate the Riemann sum for $f(x) = x^3 - 6x$ taking the sample points to the right end points and a=0, b=3 and n=6.

5. Set up an expression for $\int_1^3 e^x dx$ as a limit of sums

6. Evaluate the following integrals by interpreting each in terms of areas:

$$a)\int_0^1 \sqrt{1-x^2}\,dx \quad b)\int_0^3 (x-1)\,dx$$

7. Use the Midpoint Rule with n = 5 to approximate $\int_1^2 \frac{1}{x}$.

8. It is known that $\int_0^{10} f(x)\,dx = 17$ and $\int_0^8 f(x)\,dx = 12$, find $\int_0^8 f(x)\,dx$.

9. Using properties of definite integrals estimate $\int_0^1 e^{-x^2} dx$.

10. Find the derivative of the function $g(x) = \int_0^x \sqrt{1+t^2}\,dt$

11. Find $\frac{d}{dx}\int_1^{x^2} \sec t\,dt$

12. Evaluate the integral $\int_1^3 e^x dx$

13. Find the area under the cosine curve from 0 to b, where $0 \le b \le \frac{\pi}{2}$.

Use Part I of Fundamental Theorem of Calculus to find the derivative of the function

a) $g(x) = \int_x^{\pi} \sqrt{1+\sec t}\, dt$ b) $y = \int_{\sin x}^{1} \sqrt{1+t^2}\, dt$ c) $y = \int_{1-3x}^{1} \frac{u}{1+u^2} du$

14.Evaluate the integrals: a) $\int_0^{\frac{\pi}{4}} \sec^2 \theta \, d\theta$ b) $\int_1^{18} \sqrt{\frac{3}{z}}\, dz$

c) $\int_0^{\pi} f(x)\, dx$,where $f(x) = \begin{cases} \sin x \text{ if } 0 \le x < \frac{\pi}{2} \\ \cos x \text{ if if } \frac{\pi}{2} \le x \le \pi \end{cases}$

15. Evaluate the following:

1. $\int \left(10x^4 - 2\sec^2 x\right) dx$
2. $\int \frac{\cos\theta}{\sin^2\theta} d\theta$
3. $\int \left(x^3 - 6x\right) dx$
4. $\int_0^2 \left(2x^3 - 6x + \frac{3}{x^2+1}\right) dx$
5. $\int_1^9 \left(\frac{2t^2 + t^2\sqrt{t} - 1}{t^2}\right) dt$
6. $\int_0^2 |2x - 1| dx$
7. $\int_0^{\frac{3\pi}{2}} |\sin x|\, dx$
8. $\int_{-1}^{2} (x - 2|x|) dx$
9. $\int_{-10}^{10} \frac{2e^x}{\sinh x + \cosh x} dx$
10. $\int_1^{64} \left(\frac{1+\sqrt{x}}{\sqrt{x}}\right) dx$
11. $\int_0^1 x\left(\sqrt[3]{x} + \sqrt[4]{x}\right) dx$
12. $\int_0^1 \left(x^{10} + 10^x\right) dx$
13. $\int_0^4 |\sqrt{x} - 1|\, dx$

16. Evaluate:

1. $\int x^3 \cos(x^4+2)dx$ 2. $\int \sqrt{2x+1}\, dx$ 3. $\int \frac{xdx}{\sqrt{1-4x^2}}$ 4. $\int e^{5x} dx$

5. $\int \sqrt{1+x^2}\, x^5 dx$ 6. $\int \tan x\, dx$

7. $\int_0^4 \sqrt{2x+1}\, dx$ 8. $\int_1^2 \frac{dx}{(3-5x)^2}$ 9. $\int_1^e \frac{\ln x}{x} dx$

10. $\int 5^t \sin(5^t) dt$ 11. $\int \frac{2^t}{2^t+3} dt$ 12. $\int \frac{x}{1+x^4} dx$

13. $\int \frac{dx}{\sqrt{1-x^2}\,\sin^{-1}x}$ 14. $\int \frac{1+x}{1+x^2} dx$ 15. $\int_0^1 dx$

16. $\int_0^{\frac{\pi}{2}} \cos x \sin(\sin x)\, dx$ 17. $\int \frac{dt}{\cos^2 t \sqrt{1+\tan t}}$ 18. $\int_1^2 x\sqrt{x-1}\, dx$

19. $\int \frac{1}{x^2+a^2} dx$ 20. $\int \frac{1}{\sqrt{a^2-x^2}} dx$

17. Evaluate:

1. $\int_{-2}^{2} \left(x^6+1\right) dx$ 2. $\int_{-1}^{1} \frac{\tan x}{1+x^2+x^4} dx$

3. $\int_{-\pi/4}^{\pi/4} \left(x^3+x^4\tan x\right) dx$ 4. $\int_{-\pi/3}^{\pi/3} x^4 \sin x\, dx$

18. Evaluate 1. $\int x \sin x\, dx$ 2. $\int \ln x\, dx$ 3. $\int t^2 e^t dt$

4. $\int e^x \sin x\, dx$ 5. $\int \tan^{-1} x\, dx$ 6. $\int (x^2-2x)\cos x\, dx$ 7. $\int_0^{\frac{1}{2}} xcox\pi x dx$ 8. $\int_4^9 \frac{\ln y}{\sqrt{y}} dy$

PART B

19 Obtain the reduction formulas for the following

1. $\int \sin^n x\, dx$ 2. $\int \cos^n x\, dx$ 3. $\int \tan^n x dx$

4. $\int x^n e^x dx$ 5. $\int (\ln x)^n dx$

20. Evaluate: 1. $\int \cos^3 x\,dx$ 2. $\int \sin^5 x \cos^2 x\,dx$ 3. $\int_0^{\pi} \sin^2 x\,dx$ 4. $\int \sin^4 x\,dx$ 5. $\int \tan^6 x \sec^4 x\,dx$

6. $\int \tan^5 x \sec^7 x\,dx$ 7. $\int \sec x\,dx$ 8. $\int \frac{1-\tan^2 x}{\sec x}dx$ 9. $\int x \tan^2 x\,dx$

10. If $\int_0^{\frac{\pi}{4}} \tan^6 x \sec x\,dx = I$, express the value of $\int_0^{\frac{\pi}{4}} \tan^8 x \sec x\,dx$ in terms of I

21. Evaluate 1. $\int \frac{\sqrt{9-x^2}}{x^2}dx$ 2. $\int \sqrt{a^2-x^2}\,dx$ 3. $\int \frac{1}{x^2\sqrt{x^2+4}}dx$ 4. $\int \frac{x}{\sqrt{x^2+4}}dx$

5. $\int \frac{dx}{\sqrt{x^2-a^2}}$, $(a>0)$ 6. $\int_0^{\frac{3\sqrt{3}}{2}} \frac{x^3}{(4x^2+4)^{\frac{3}{2}}}dx$ 7. $\int \frac{t^5}{\sqrt{t^2+2}}dx$

8. $\int \frac{x^2}{(x^2+a^2)^{\frac{3}{2}}}dx$ a) by trigonometric substitution b) by the hyperbolic substitution x= asinht.

22. Evaluate 1. $\int \frac{x^3+x}{x-1}dx$ 2. $\int \frac{x^2+2x-1}{2x^3+3x^2-2x}dx$ 3. $\int \frac{dx}{x^2-a^2}$. 4. $\int \frac{x^4-2x^2+4x-1}{x^3-x^2-x+1}dx$

5. $\int \frac{2x^2-x+4}{x^3+4}dx$ 6. $\int \frac{4x^2-3x+2}{4x^2-4x+3}dx$ 7. $\int \frac{1-x+2x^2-x^3}{x(x^2+1)^2}dx$ 8. $\int \frac{dx}{1-\cos x}$.

23. Determine whether the integral $\int_1^{\infty} \frac{1}{x}dx$ is convergent or divergent.

24. Evaluate $\int_t^0 xe^x\,dx$

25. Evaluate $\int_{-\infty}^{\infty} \frac{1}{1+x^2}dx$

26. For what values of p is the integral $\int_1^{\infty} \frac{1}{x^p}dx$ convergent?

27. Find $\int_2^5 \frac{1}{\sqrt{x-2}}dx$

28. Find $\int_{2}^{5} \frac{1}{\sqrt{x-2}} dx$

29. Evaluate $\int_{0}^{3} \frac{dx}{x-1}$,if possible

30.Evaluate $\int_{0}^{1} \ln x \, dx$

31. Show that $\int_{0}^{\infty} e^{-x^2} dx$ is convergent.

32. $\int_{1}^{\infty} \frac{1+e^{-x}}{x} dx$ is divergent

34. Determine whether each integral is convergent or divergent. Evaluate those that are convergent

1. $\int_{3}^{\infty} \frac{dx}{(x-2)^{3/2}}$ 2. $\int_{-\infty}^{0} \frac{1}{3-4x} dx$ 3. $\int_{2}^{\infty} e^{-5p} dp$ 4. $\int_{0}^{\infty} \frac{x^2}{\sqrt{1+x^3}} dx$

5. $\int_{-\infty}^{\infty} \frac{x^2}{9+x^6} dx$ 6. $\int_{0}^{1} \frac{\ln x}{\sqrt{x}} dx$ 7. $\int_{0}^{3} \frac{dx}{x^2-6x+5}$ 8. $\int_{0}^{\infty} \frac{1}{\sqrt{x}(1+x)} dx$

35. Use the comparison theorem to determine whether the integral is convergent or divergent.

1. $\int_{1}^{\infty} \frac{2+e^{-x}}{x} dx$ 2. $\int_{0}^{\pi} \frac{\sin^2 x}{\sqrt{x}} dx$

UNIT IV MULTIPLE INTEGRALS
PART A

1. Evaluate the following:

1. $\int_0^1 \int_x^{\sqrt{x}} xy\,(x+y)\,dx\,dy$

2. $\int_0^{\pi/2} \int_0^{\sin\theta} r\,d\theta\,dr$

3. $\int_0^{\log 2} \int_0^{x} \int_0^{x+y} e^{x+y+z}\,dx\,dy\,dz$

$$\int_0^1 \int_0^{\sqrt{(1-x^2)}} \int_a^{\sqrt{1-x^2-y^2}} \frac{dx\,dy\,dz}{\sqrt{1-x^2-y^2-z^2}}$$

PART B

2. Evaluating the given double integral over the given plane region :

Problems:

1. Evaluate $\iint (x-y)\,dx\,dy$ over the region bounded by the line $y = x$ and the parabola $y = x^2$
2. Evaluate $\iint_R \frac{e^{-y}}{y}\,dx\,dy$ where R is the region bounded by the lines $x = 0, x = y$ **and** $y = \infty$ s
3. Evaluate $\iint xy\,dx\,dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$.
4. Evaluate $\iint x\,dx\,dy$ over the positive quadrant of the circle $x^2 - 2ax + y^2 = 0$.
5. Evaluate $\iint (x^2 + y^2)\,dx\,dy$ over the region bounded by the parabola $y^2 = 4x$ and its latus rectum.

3. Evaluating the given triple integral over the given solid region:

1. Evaluate $\iiint_V \frac{dz\,dy\,dx}{(x+y+z+1)^3}$ where V is the region bounded by $x = 0$, $y = 0$, $z = 0$ **and** $x + y + z = 1$.
2. Find the value of $\iiint xyz\,dx\,dy\,dz$ through the positive spherical octant for which $x^2 + y^2 + z^2 = a^2$
3. Evaluate $\iiint x^2 yz\,dx\,dy\,dz$ taken over the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$ **and** $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

4. Change the order of integration and evaluate:

Problems:

1. $\int_0^1 \int_y^1 \frac{x}{x^2+y^2}\, dx\, dy$

2. $\int_0^2 \int_0^{\sqrt{2x-x^2}} (x^2+y^2)\, dy\, dx$

3. $\int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx$

4. $\int_1^4 \int_{2/y}^{2\sqrt{y}} dx\, dy$

5. $\int_0^a \int_{x^2/a}^{2a-x} xy\, dy\, dx$

6. $\int_1^3 \int_0^{6/x} x^2 y\, dy\, dx$

7. $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dy\, dx$

8. $\int_0^a \int_0^{\frac{b}{a}\sqrt{a^2-x^2}} x^2\, dy\, dx$

$\int_0^a \int_{a-y}^{\sqrt{a^2-y^2}} y\, dx\, dy$

5. Problems:

1. Find, by double integration, the area enclosed by the curves $y^2 = 4ax$ and $x^2 = 4ay$
2. Find the area between the curves $y^2 = 9x$ and $x^2 = 9y$.
3. Find the area between the curves $y^2 = 4ax$ and x + y = 3a.
4. Find the area enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ using double integration.
5. Find the area common to the parabola $y^2 = x$ and $x^2 + y^2 = 2$.
6. Find the area bounded by the parabola $y^2 = 4 - x$ and $y^2 = x$ by double integration,
7. Find the area between the circle $x^2 + y^2 = a^2$ and the line x + y = a lying in the first quadrant by double integration.

8. Evaluate $\iint_S (y + 2z - 2)\, ds$ where S is the part of the plane 2x + 3y + 6z = 12 that lies in the positive octant.

9. Evaluate $\iint_S z^3\, ds$ where S is the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.

10. Evaluate $\iint_S y(z+x)\,ds$ where S is the curved surface of the cylinder $x^2+y^2=16$ that lies in the positive octant and that is included between the planes z = 0 and z = 5.

11. Find the area of the cardiod $r=a(1+\cos\theta)$

12. Find the area enclosed by the curve $r^2=a^2\cos 2\theta$ by double integration

13. Find the area inside the circle $r=a\sin\theta$ and lying outside the cardiod $r=a(1-\cos\theta)$.

14. Find the area that lies inside the cardiod $r=a(1+\cos\theta)$ and outside the circle r = a, by double integration

15. Find the area that lies outside the circle $r=a\cos\theta$ and inside the circle $r=2a\cos\theta$

16. By transforming in to polar coordinates evaluate the following double integrals:

1. $\iint \frac{xy}{x^2+y^2}\,dx\,dy$ taken over the annular region between the circles $x^2+y^2=4$ and $x^2+y^2=16$.

2. $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$ and hence evaluate $\int_0^\infty e^{-x^2}\,dx$.

3. $\int_0^a\int_y^a \frac{x^2}{(x^2+y^2)^{3/2}}\,dx\,dy$

4. $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}} \frac{x}{\sqrt{x^2+y^2}}\,dx\,dy$

5. $\int_0^a\int_0^{\sqrt{a^2-x^2}} e^{-(x^2+y^2)}\,dx\,dy$

6. $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{dx\,dy}{(a^2+x^2+y^2)^{3/2}}$

UNIT V DIFFERENTIAL EQUATIONS
PART A

1. - Problems on higher order linear differential equations with constant coefficients:

1. Solve: $(D^2-3D+2)y = 2e^x + 2\cos(2x + 3)$

2. Solve: $(D^2+3D+2)y = \sin^2 x + x^2 + 2x + 7$

3. Solve: $(D^2+4)y = \cos^3 x$

4. Solve: $(D^2+5D+4)y = e^{-x} \sin 2x$

5. Solve: $(D^2+4D+3)y = 6\, e^{-2x} \sin x \sin 2x$

6. Solve: $(D^2-2D+1)y = 8\, x\, e^x \sin x$

7. Solve: $(D^2+ a^2)y = \cosh ax$

8. Solve: $\left(D^4 + 6D^3 + 11D^2 + 6D\right) y = 20\, e^{-2x} \sin x$

PART B

2. - Problems on method of variation of parameters:

1. Solve: $(2D^2+8)y = \tan 2x$

2. Solve: $(D^2+ a^2)y = \sec ax$

3. Solve: $(D^2+ 4)y = \cot 2x$

4. Solve: $(D^2+ 1)y = \operatorname{cosec} x$

5. Solve: $(D^2+ 2D+ 1)y = \dfrac{e^{-x}}{x^2}$

6. Solve: $(D^2+ 1)y = x \sin x$

3. - Problems on linear differential equations with variable coefficients:

1. Solve: $(x^2 D^2 - 2xD - 4)\, y = x^2 + 2\log x$

2. Solve $\dfrac{d^2y}{dx^2} + \dfrac{1}{x}\dfrac{dy}{dx} = \dfrac{12\log x}{x^2}$

3. Solve $x^2\dfrac{d^2y}{dx^2} - 3x\dfrac{dy}{dx} + 4y = x^2 \log x$

4. Solve: $(x^2 D^2 - xD + 4)\, y = x^2 \sin(\log x)$

5. Solve: $(x^2 D^2 + 3xD + 5)\, y = x \cos(\log x)$

6. Solve $x^2 \dfrac{d^2y}{dx^2} + 4x\dfrac{dy}{dx} + 2y = x^2 + \dfrac{1}{x^2}$

7. Solve $(3x+2)^2 \frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1$

8. Solve $(x+1)^2 \frac{d^2y}{dx^2} + (x+1)\frac{dy}{dx} + y = 4\cos\log(x+1)$

4. - Problems on simultaneous differential equations with constant coefficients:

1. Solve $\frac{dx}{dt} + 2y = \sin 2t$, $\frac{dy}{dt} - 2x = \cos 2t$.

2. Solve (D + 2) x − 3y = t-3x + (D + 2) y = e^{2t} where D = d/dt

3. Sove: $\frac{dx}{dt} = 3x + 8y$ $\frac{dy}{dt} = -x - 3y$