DHANALAKSHMI SRINIVASAN COLLEGE OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF MATHEMATICS

QUESTION BANK

V SEMESTER

III SEMESTER

MA8353- TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

Regulation – 2017

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Academic Year 2018- 2019

QUESTION BANK

SUBJECT : MA8353- TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

SEMESTER / YEAR: III / II (CIVIL, EEE, EIE & MECH)

UNIT I - PARTIAL DIFFERENTIAL EQUATIONS

Formation of partial differential equations – Singular integrals – Solutions of standard types of first order partial differential equations – Lagrange's linear equation – Linear partial differential equations of second and higher order with constant coefficients of both homogeneous and non-homogeneous types.

PART- A

Q.No.	Question	Bloom's Taxonomy Level	Domain
1.	Form a partial differential equation by eliminating the arbitrary constants 'a' and 'b' from $z = ax^2 + by^2$.	BTL -6	Creating
2.	Eliminate the arbitrary function from $z = f(x^2 - y^2)$ and form the partial differential equation	BTL -6	Creating
3.	Construct the partial differential equation of all spheres whose centers lie on the x-axis.	BTL -3	Applying
4.	Form the partial differential equation by eliminating the arbitrary function f from $z = e^{ay} f(x + by)$.	BTL- 6	Creating
5.	Form the partial differential equation by eliminating the arbitrary constants a, b from the relation $\log(az - 1) = x + ay + b$.	BTL -6	Creating
6.	Form the PDE by eliminating the arbitrary function from $\phi\left[z^2 - xy, \frac{x}{z}\right] = 0$	BTL -6	Creating
7.	Form the partial differential equation from $(x-a)^2 + (y-b)^2 = z^2 \cot^2 \alpha$	BTL -6	Creating
8.	Form the partial differential equation by eliminating the arbitrary function Φ from $\Phi(x^2 - y^2, z) = 0$	BTL -6	Creating
9.	Form the partial differential equation by eliminating arbitrary constants a and b from $(x-a)^2 + (y-b)^2 + z^2 = 1$	BTL -6	Creating
10.	Find the complete integral of $\sqrt{p} + \sqrt{q} = 1$.	BTL -3	Applying
11.	Find the complete solution of $q = 2px$	BTL -3	Applying
12.	Find the complete integral of $p + q = pq$	BTL -3	Applying
13.	Solve $px^2 + qy^2 = z^2$	BTL -3	Applying

14.	Solve $(D^2 - 7DD' + 6D'^2)z = 0$	BTL -3	Applying
15.	Solve $(D^3 - D^2 D' - 8DD'^2 + 12D'^3)z = 0$	BTL -3	Applying
16.	Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial x} = 0$	BTL -3	Applying
17.	Solve $(D^4 - D'^4)z = 0$	BTL -3	Applying
18.	Solve $(D + D' - 1)(D - 2D' + 3)z = 0$	BTL -3	Applying
19.	Solve $(D - D')^3 z = 0$	BTL -3	Applying
20.	Solve $(D - 1)(D - D' + 1)z = 0$	BTL -3	Applying
PART – B			
1.(a)	Find the PDE of all planes which are at a constant distance 'k' units from the origin.	BTL -6	Creating
1. (b)	Find the singular integral of $z = px + qy + 1 + p^2 + q^2$	BTL -2	Understanding
2. (a)	Form the partial differential equation by eliminating arbitrary function Φ from $\Phi(x^2 + y^2 + z^2, ax + by + cz) = 0$	BTL -6	Creating
2.(b)	Find the singular integral of $z = px + qy + p^2 + pq + q^2$	BTL -2	Understanding
3. (a)	Form the partial differential equation by eliminating arbitrary functions f and g from $z = x f(\frac{y}{x}) + y g(x)$	BTL -6	Creating
3.(b)	Solve $(\frac{p}{2} + x)^2 + (\frac{q}{2} + y)^2 = 1\,2$	BTL -3	Applying
4. (a)	Solve $x^2 p + y^2 q = z(x + y)$	BTL -3	Applying
4.(b)	Form the partial differential equation by eliminating arbitrary function f and g from the relation $z = x f(x + t) + g(x + t)$	BTL -6	Creating
5. (a)	Solve $(x^2 - yz)p + (y^2 - xz)q = (z^2 - xy)$	BTL -3	Applying
5.(b)	Solve $9(p^2 z + q^2) = 4$	BTL -3	Applying
6. (a)	Find the general solution of $(3z - 4y)p + (4x - 2z)q = 2y - 3x$	BTL -2	Understanding
6.(b)	Solve $(y^2 + z^2)p - xyq + xz = 0$	BTL -3	Applying

7. (a)	Find the complete solution of $z^2(p^2+q^2+1)=1$	BTL -4	Analyzing
7. (b)	Find the general solution of $(D^2+2DD'+D'^2)z=2\cos y-x\sin y$	BTL -2	Understanding
8. (a)	Find the general solution of $(D^2+D'^2)z=x^2y^2$	BTL -2	Understanding

8.(b)	Find the singular integral of $z = px + qy + p^2 - q^2$	BTL -2	Understanding
9. (a)	Solve $(D^2 - 3DD' + 2D'^2)\, z = (2 + 4x)e^{x+2y}$	BTL -3	Applying
9.(b)	Find the general solution of $(z^2 - y^2 - 2yz)\, p + (xy + zx)q = (xy - zx)$	BTL -2	Understanding
10.(a)	Solve $x(y^2 - z^2)\, p + y(z^2 - x^2)q = z(x^2 - y^2)$	BTL -3	Applying
10.(b)	Solve $(D^2 - 3DD' + 2D'^2)z = \sin(x + 5y)$	BTL -3	Applying
11.(a)	Solve the Lagrange's equation $(x + 2z)\, p + (2xz - y)q = x^2 + y$	BTL -3	Applying
11.(b)	Solve $(D^2 - DD' - 2D'^2)z = 2x + 3y + e^{2x+4y}$	BTL -3	Applying
12(a)	Solve $(D^2 - 5DD' + 6D'^2)z = y \sin x$	BTL -3	Applying
12.(b)	Solve the partial differential equation $(x - 2z)\, p + (2z - y)\, q = y - x$	BTL -3	Applying
13.(a)	Solve $(D^2 - DD' - 20D'^2)\, z = e^{5s+y} + \sin(4x - y)$.	BTL -3	Applying
13.(b)	Solve $(D^2 - 3DD' + 2D'^2 + 2D - 2D')z = \sin(2x + y)$.	BTL -3	Applying
14.(a)	Solve $(D^2 + 2DD' + D'^2)z = x^2 y + e^{x-y}$	BTL -3	Applying
14.(b)	Solve $(D^3 - 7DD'^2 - 6D'^3)z = \sin(x + 2y)$	BTL -3	Applying

UNIT II FOURIER SERIES:

Dirichlet's conditions – General Fourier series – Odd and even functions – Half range Sine series – Half range Cosine series – Complex form of Fourier series – Parseval's Identity – Harmonic analysis.

PART –A

Q.No	**Question**	**Bloom's Taxonomy Level**	**Domain**
1.	State the Dirichlet's conditions for a function f(x) to be expanded as a Fourier series.	**BTL -1**	Remembering
2.	State the sufficient condition for a function f(x) to be expressed as a Fourier Series.	**BTL -1**	Remembering
3.	If $(\pi - x)^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$ in $0 < x < 2\pi$ then deduce that value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$.	**BTL -1**	Remembering
4.	Does $f(x) = \tan x$ posses a Fourier expansion?	**BTL -2**	Understanding
5.	Determine the value of a_n in the Fourier series expansion of	**BTL -5**	Evaluating

	$f(x) = x^3$ in $(-\pi, \pi)$.		
6.	Find the constant term in the expansion of $\cos^2 x$ as a Fourier series in the interval $(-\pi, \pi)$.	**BTL -2**	Understanding
7.	If f(x) is an odd function defined in (-l, l). What are the values of a_0 and a_n?	**BTL -2**	Understanding
8.	If the function f(x) = x in the interval $0<x<2\pi$ then find the constant term of the Fourier series expansion of the function f.	**BTL -2**	Understanding
9.	If the Fourier series of the function $f(x) = x+x^2$, $-\pi < x < \pi$. with Period 2π is given by $f(x) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} (-1)^n \left[\frac{4}{n^2} \cos nx - \frac{2}{n} \sin nx \right]$, then find the value of the infinite series $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$	**BTL -4**	Analyzing
10.	Write a_0 , a_n in the expression $x + x^3$ as a Fourier series in $(-\pi, \pi)$	**BTL -3**	Applying
11.	Write the Complex form of Fourier Series for a function f(x) defined in $-l \leq x \leq l$.	**BTL -3**	Applying
12.	Find the root mean square value of $f(x) = x^2$ in $(0,\pi)$	**BTL -1**	Remembering
13.	Find the RMS value of $f(x) = x(l - x)$ in $0 \leq x \leq l$	**BTL -3**	Applying
14.	Find the RMS value of $f(x) = x^2$ in (0, l)	**BTL -1**	Remembering
15.	Write down the Parseval's formula on Fourier coefficients	**BTL -5**	Evaluating
16.	Define the RMS value of a function f(x) over the interval (a, b)	**BTL -6**	Creating
17.	Without finding the values of a_0 , a_n and b_n of the Fourier series, for the function $f(x) = x^2$ in the interval $(0,2\pi)$ find the value of $\left\{\frac{a_0^2}{2} + \sum_{n=1}^{\infty} (a_n^2 + b_n^2)\right\}$	**BTL -4**	Analyzing
18.	Find the R.M.S value of $f(x) = 1 - x$ in $0 < x < 1$.	**BTL -1**	Remembering
19.	State Parseval's identity for the half-range cosine expansion of f(x) in (0,1).	**BTL -6**	Creating
20.	What is meant by Harmonic Analysis?	**BTL -3**	Applying
	PART – B		

1.(a)	Find the Fourier series expansion of	**BTL -1**	Remembering

	$f(x) = \begin{cases} x & \text{for } 0 \le x \le 1 \\ 2-x & \text{for } 1 \le x \le 2. \end{cases}$		
1.(b)	Find the Fourier series of $f(x) = x^2$ in $-\pi < x < \pi$.Hence deduce the value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$	**BTL -1**	Remembering
2.(a)	Obtain the Fourier series to represent the function $f(x) = \lvert x\rvert, -\pi < x < \pi$ and Deduce $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$.	**BTL -2**	Understanding
2.(b)	Find the Fourier series of the function $f(x) = \begin{cases} 0 & -\pi \le x \le \pi \\ \sin x & 0 \le x \le \pi \end{cases}$ and Hence Evaluate $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots\ldots$	**BTL -2**	Understanding
3.(a)	Expand $f(x) = \begin{cases} 1 + \frac{2s}{\pi}, & -\pi < x < 0 \\ 1 - \frac{2s}{\pi}, & 0 < x < \pi \end{cases}$ as a full range Fourier series in the interval $(-\pi, \pi)$.Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \infty = \frac{\pi^2}{8}$.	**BTL -1**	Remembering
3.(b)	Find a Fourier series with period 3 to represent $f(x) = 2x - x^2$ in $(0,3)$.	**BTL -1**	Remembering
4.(a)	Determine the Fourier series for the function $f(x) = x\sin x$ in $0 < x < 2\pi$.	**BTL -5**	Evaluating
4.(b)	Obtain the Fourier series for the function f(x) given by $f(x) = \begin{cases} 1 - x, & -\pi < x < 0 \\ 1 + x, & 0 < x < \pi \end{cases}$ Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$	**BTL -3**	Applying
5.(a)	Determine the Fourier series for the function $f(x) = \lvert\cos x\rvert$ in $-\pi < x < \pi$.	**BTL -5**	Evaluating
5.(b)	Expand $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \end{cases}$ as a series of cosines in the interval (0,2).	**BTL -1**	Remembering

6.(a)	Find the Fourier expansion of the following periodic function of period 4 $f(x) = \begin{cases} 2+x, & -2 \le x \le 0 \\ 2-x, & 0 \le x \le 2 \end{cases}$	**BTL -2**	Remembering

	Hence deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\infty=\frac{\pi^2}{8}$.		
6.(b)	Find the half range sine series of $f(x) = 4x - x^2$ in the interval (0,4).Hence deduce the value of the series $\frac{1}{1^3}-\frac{1}{3^3}+\frac{1}{5^3}-\frac{1}{7^3}+\cdots\infty$.	**BTL -2**	Remembering
7.(a)	Find the Fourier series of $f(x) = \lvert sinx\rvert\ in -\pi < x < \pi$ of periodicity 2π.	**BTL -4**	Analyzing
7.(b)	Find the Fourier series of $f(x) = x + x^2$ in ($-\pi, \pi$) with period $2\ \pi$. Hence deduce $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$.	**BTL -3**	Applying
8.(a)	Compute the first two harmonics of the Fourier series of f(x) from the table given	**BTL -6**	Creating
8.(b)	Find the complex form of the Fourier series of Cosax in $(-\pi, \pi)$ where "a" is not an integer.	**BTL -2**	Remembering
9.(a)	Find the Fourier series of $f(x) = x^2\ in\ (-\pi, \pi)$ and hence deduce that $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+\cdots\infty=\frac{\pi^4}{90}$	**BTL -2**	Remembering
9.(b)	(see table below) The above table gives the variation of a periodic current over a period by harmonic analysis, Show that there is a direct current part of 0.75amps in the variable current. Also obtain the amplitude of the first harmonic.	**BTL -3**	Applying

t secs	0	T/6	T/3	T/2	2T/3	5T/6	T
A amps	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

10.(a)	Find the half range Fourier cosine series of $f(x) = (\pi - x)^2$ in the interval $(0, \pi)$. Hence Find the sum of the series $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots\ldots$	**BTL -1**	Remembering
10.(b)	Obtain the fourier cosine series expansion of $f(x) = x$ $in\ \ 0 < x < 4$.Hence deduce the value of $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots\ \infty$.	**BTL -1**	Remembering
11.(a)	By using Cosine series $show\ that$ $\frac{\pi^4}{96} = 1 + \frac{1}{3^4} + \frac{1}{5^4} + \cdots$ for $f(x) = x\ in\ 0 < x < \pi$	**BTL -4**	Analyzing

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
f(x)	1	1.4	1.9	1.7	1.5	1.2	1

<table>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>11.(b)</td><td>Find the Fourier cosine series up to third harmonic to represent the function given by the following data:

x	0	1	2	3	4	5
y	4	8	15	7	6	2

</td><td>BTL -6</td><td>Creating</td></tr>
<tr><td>12.(a)</td><td>Show that the complex form of Fourier series for the function
$f(x) = e^{s} when - \pi < x < \pi$ and $f(x) =$
$f(x + 2\pi)\ \ is\ \ f(x) = \frac{cinh\pi}{\pi}\sum_{n=-\infty}^{\infty}(-1)^n\frac{1+in}{1+n^2}e^{ins}$.</td><td>BTL -1</td><td>Remembering</td></tr>
<tr><td>12.(b)</td><td>Find the complex form of the Fourier series of
$f(x) = e^{as}\, in - \pi < x < \pi.$</td><td>BTL -4</td><td>Analyzing</td></tr>
<tr><td>13.</td><td>Calculate the first 3 harmonics of the Fourier of f(x) from the following data

x	0	30	60	90	120	150	180	210	240	270	300	330	360
f(x)	1.8	1.1	0.3	0.16	0.5	1.3	2.16	1.25	1.3	1.52	1.76	2	1.8

</td><td>BTL -6</td><td>Creating</td></tr>
<tr><td>14.(a)</td><td>Find the complex form of the Fourier series of
$f(x) = e^{-as}\, in - l < x < l.$</td><td>BTL -4</td><td>Analyzing</td></tr>
<tr><td>14.(b)</td><td>Find the Fourier series as far as the second harmonic to represent the function $f(x)$ With period 6

x	0	1	2	3	4	5
y	9	18	24	28	26	20

</td><td>BTL -6</td><td>Creating</td></tr>
</table>

UNIT III -APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS

Classification of PDE – Method of separation of variables – Fourier Series solutions of one dimensional wave equation - One dimensional equation of heat conduction – Steady state solution of two dimensional equation of heat conduction.

PART -A

Q.No.	**Question**	**Bloom's Taxonomy Level**	**Domain**
1.	Classify the PDE $(1 - x^2)Z_{ss} - 2xyZ_{sy} + (1 - y^2)Z_{yy} + xZ_s + 3x^2yz - 2Z = 0$	BTL-4	Analyzing
2.	Classify the PDE $u_{ss} + u_{sy} = f(x, y)$	BTL-4	Analyzing

3.	Solve $\frac{\partial u}{\partial s} = 2\frac{\partial u}{\partial t} + u$ where $u(x, 0) = 6e^{-3s}$ by method of separation of variables	BTL-3	Applying
4.	What are the various solutions of one dimensional wave equation	BTL-1	Remembering
5.	In the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ what does C^2 stand for?	BTL-2	Understanding
6.	What is the basic difference between the solutions of one dimensional wave equation and one dimensional heat equation with respect to the time?	BTL-3	Applying
7.	Write down the initial conditions when a taut string of length 2lis fastened on both ends. The midpoint of the string is taken to a height b and released from the rest in that position	BTL-1	Remembering
8.	A slightly stretched string of length l has its ends fastened at x = 0 and x = l is initially in a position given by $y(x, 0) = y_0 sin^3 \frac{\pi s}{l}$.If it is released from rest from this position, write the boundary conditions	BTL-2	Understanding
9.	A tightly stretched string with end points $x = 0$&$x = l$is initially at rest in equilibrium position. If it is set vibrating giving each point velocity$\lambda x(l - x)$. Write the initial and boundary conditions	BTL-2	Understanding
10.	If the ends of a string of length l are fixed at both sides. The midpoint of the string is displaced transversely through a height h and the string is released from rest, state the initial and boundary conditions	BTL-2	Understanding
11.	State the assumptions in deriving the one dimensional heat equation	BTL-1	Remembering
12.	Write down the various possible solutions of one dimensional heat flow equation?	BTL-1	Remembering
13.	In the one dimensional heat equation $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$ what is C^2 ?	BTL-2	Understanding
14.	The ends A and B of a rod of length 20 cm long have their temperature kept 30^0 C and 80^0 C until steady state prevails. Find the steady state temperature on the rod	BTL-2	Understanding
15.	An insulated rod of length 60 cm has its ends at A and B maintained at 20^0C and 80^0 C respectively. Find the steady state solution of the rod.	BTL-2	Understanding
16.	An insulated rod of length l cm has its ends at A and B maintained at 0^0C and 80^0 C respectively. Find the steady state solution of the rod.	BTL-2	Understanding
17.	Write down the three possible solutions of Laplace equation in two dimensions	BTL-1	Remembering
18.	Write down the governing equation of two dimensional steady state heat equation.	BTL-1	Remembering

19.	A rectangular plate with insulated surface is 10cm wide. The two long edges and one short edge are kept at0^0 C, while the	BTL-2	Understanding

	temperature at short edge x =0 is given by $u = \begin{cases} 20y, & 0 \le y \le 5 \\ 20(10-y), & 5 \le y \le 10 \end{cases}$ Find the steady state temperature at any point in the plate.		
20.	A plate is bounded by the lines x=0, y=0, x=l and y=l. Its faces are insulated. The edge coinciding with x-axis is kept at 100^0 C. The edge coinciding with y-axis at 50^0 C. The other 2 edges are kept at 0^0 C. write the boundary conditions that are needed for solving two dimensional heat flow equation.	BTL-2	Understanding
PART-B			
1.	A string is stretched and fastened to two points that are distinct string *l* apart. Motion is started by displacing the string into the form$y = k(lx - x^2)$from which it is released at time t=0. Find the displacement of any point on the string at a distance of x from one end at time t.	BTL-2	Understanding
2.	A tightly stretched string of length 2 *l* is fastened at both ends. The Midpoint of the string is displaced by a distance b transversely and the string is released from rest in this position. Find an expression for the transverse displacement of the string at any time during the subsequent motion.	BTL-2	Understanding
3.	A slightly stretched string of length *l* has its ends fastened at $x = 0$and$x = l$ is initially in a position given by $y(x,0) = y_0 sin^3 \frac{\pi s}{l}$.If it is released from rest from this position, find the displacement y at any distance xfrom one end and at any time.	BTL-2	Understanding
4.	A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If it is set vibrating string giving each point a velocity $\lambda x(l - x)$. Find the displacement of the string at any distance x from one end at any time t.	BTL-3	Applying
5.	A tightly stretched string with fixed end points x=0 and x=*l* is initially at rest in its equilibrium position. If it is vibrating string by giving to each of its points a velocity $v = \begin{cases} \frac{2cx}{l} & if\, 0 \le x \le \frac{l}{2} \\ \frac{2c(l-x)}{l} & if\, \frac{l}{2} \le x \le l \end{cases}$. Find the displacement of the string at any distance x from one end at any time t.	BTL-2	Understanding
6.	A tightly stretched string of length l is initially at rest in this equilibrium position and each of its points is given the velocity $v_0 \sin^3 \frac{\pi x}{l}$. Find the displacement y(x, t).	BTL-2	Understanding
7.	Solve$\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$subject to the conditions (i) u(0,t)=0	BTL-3	Applying

	for all $t \le 0$ (ii) $u(l, t) = 0$ for all $t \le 0$ (iii) $u(x,0) = \begin{cases} x & if\ 0 \le x \le \frac{l}{2} \\ l - x & if\ \frac{l}{2} \le x \le l \end{cases}$.		
8.	A rod 30 cm long has its ends A and B kept at 20^0 and 80^0 respectively until steady state conditions prevail. The temperature at each end is then suddenly reduced to 0^0C and kept so. Find the resulting temperature function u(x, t) taking x = 0 at A.	BTL-2	Understanding
9.	A bar 10 cm long with insulated sides has its ends A and B maintained at temperature 50^0C and 100^0C respectively. Until steady state conditions prevails.The temperature at A is suddenly raised to 90^0C and at the same time lowered to 60^0C at B. Find the temperature distributed in the bar at time t.	BTL-2	Understanding
10.	A square plate is bounded by the lines $x = 0, y = 0, x = 20$ $y = 20$. Its faces are insulated. The temperature along the upper horizontal edge is given by $u(x, 20) = x\ (20 - x)$ when $0 < x < 20$ while the other three edges are kept at 0^0 C. Find the steady state temperature in the plate.	BTL-2	Understanding
11.	A square metal plate is bounded by the lines x=0, x=a, y=0, y=a .The edges x=a, y=0, y=a are kept at zero degree temperature while the temperature at the edge x=0 is ky. Find the steady state temperature distribution at in the plate.	BTL-2	Understanding
12.	A rectangular plate with insulated surface is 10 cm wide and so long compared to its width that it may be considered infinite in length without introducing appreciable error. The temperature at short edge y=0 is given by $u = \begin{cases} 20x & , \quad 0 \le x \le 5 \\ 20(10 - x), & 5 \le x \le 10 \end{cases}$ and all the other three edges are kept at 0^0C. Find the steady state temperature at any point in the plate.	BTL-2	Understanding
13.	An infinitely long rectangular plate with insulated surface is 10cm wide. The two long edges and one short edge are kept at 0^0C, while the other short edge x=0 is kept at temperature $u = \begin{cases} 20y & , \quad 0 \le y \le 5 \\ 20(10 - y), & 5 \le y \le 10 \end{cases}$. Find the steady state temperature distribution in the plate.	BTL-2	Understanding
14.	A long rectangular plate with insulated surface is lcm . If the temperature along one short edge y=0 is $u(x,0)=K(\ l\ x - x^2)$ degrees, for $0 < x < l$, while the other 2 edges x=0 and x=l as well as the other short edge are kept at 0^0C, find the steady state temperature function $u(x, y)$.	BTL-2	Understanding

UNIT –IV FOURIER TRANSFORM

Statement of Fourier integral theorem – Fourier transform pair – Fourier sine and cosine transforms – Properties – Transforms of simple functions – Convolution theorem – Parseval's identity.

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PART –A			
Q.No.	**Question**	**Bloom's Taxonomy Level**	**Domain**
1.	State Fourier integral Theorem	BTL -1	Remembering
2.	Write Fourier Transform in Pairs	BTL -1	Remembering
3.	If $F(s)$ denote the Fourier Transform of $f(x)$, Prove that $F[f(ax)] = \frac{1}{a} F\left(\frac{s}{a}\right), a \neq 0.$	BTL -3	Applying
4.	If the Fourier Transform of $f(x)$ is $F(s) = F[f(x)]$, *then showthat* $F[f(x-a)] = e^{ias} F(s).$	BTL -3	Applying
5.	Find the Fourier Transform of $e^{-a x}$.	BTL -2	Understanding
6.	Find the Fourier Transform of $f(x) = \begin{cases} e^{i k x}, & if\, a < x < b \\ 0, & if \quad x \leq a \;\&\; x > b \end{cases}.$	BTL -2	Understanding
7.	State and Prove Modulation theorem on Fourier Transforms	BTL -2	Understanding
8.	Find the Fourier sine Transform of $3e^{-2s}$.	BTL -2	Understanding
9.	Define self-reciprocal with respect to Fourier Transform	BTL -1	Remembering
10.	Find the infinite Fourier sine Transform of $\frac{1}{x}$.	BTL -2	Understanding
11.	Find the Fourier sine Transform of $f(x) = e^{-x^2}$.	BTL -2	Understanding
12.	Give an example of a function which is self- reciprocal under Fourier Sine & Cosine Transform	BTL -3	Applying
13.	Write down the Fourier cosine Transform pair of formulae	BTL -1	Remembering
14.	If F(s) is the Fourier Transform of $f(x)$. Show that the Fourier Transform of $e^{iax} f(x) is\ F(s+a).$	BTL -3	Applying
15.	Show that the Fourier Transform of the derivatives of a function $F\left[\frac{d^n}{dx^n} f(x)\right] = (-is)^n F(s).$	BTL -3	Applying
16.	If $F(s) = F[f(x)]$, *then find* $F[x^n f(x)]$.	BTL -2	Understanding
17.	Find the Fourier cosine Transform of e^{-2x}.	BTL -2	Understanding

18.	Let $F_c(s)$ be the Fourier cosine Transform of $f(x)$. Prove that $F_c\left(f(x)\cos ax\right) = \frac{1}{2}\left[F_c(s+a) + F_c(s-a)\right]$.	BTL -3	Applying
19.	State Convolution theorem in Fourier Transform	BTL -1	Remembering

20.	State Parseval's Identity on Fourier Transform	BTL -1	Remembering
PART-B			
1.(a)	Find the Fourier Transform of $f(x)=\begin{cases}1\ , & \lvert x\rvert \le a\\ 0\ , & \lvert x\rvert > a>0\end{cases}$ and hence evaluate $\int_0^\infty \left(\frac{\sin t}{t}\right)dt$. Also using Parseval's Identity Prove that $\int_0^\infty \left(\frac{\sin^2 t}{t^2}\right)dt = \frac{\pi}{2}$	BTL -2	Understanding
1. (b)	Find the Fourier Cosine Transform of the function $f(x)=\frac{e^{-ax}-e^{-bx}}{x}\ , x>0$	BTL -2	Understanding
2. (a)	Find the Fourier Transform of the function f(x) = $\begin{cases}1-\lvert x\rvert\ , & if\ \ \lvert x\rvert \le 1\\ 0\ , & if\ \ \lvert x\rvert > 1\end{cases}$ Hence deduce that (i) $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt=\frac{\pi}{2}$ $(ii)\ \int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt=\frac{\pi}{3}$.	BTL -2	Understanding
2.(b)	Show that the function $e^{\frac{-x^2}{2}}$ is self-reciprocal under the Fourier Transform by finding the Fourier Transform of $e^{-a^2x^2}, a>0$	BTL -3	Understanding
3.	Show that the Fourier Transform of $f(x)=\begin{cases}a-\lvert x\rvert\ , & if\ \ \lvert x\rvert \le a\\ 0\ , & if\ \ \lvert x\rvert > a>0\end{cases}$ is $\frac{2}{\pi}\left(\frac{1-\cos as}{s^2}\right)$. Hence deduce that $(i)\ \int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt=\frac{\pi}{2}$, $(ii)\ \int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt=\frac{\pi}{3}$.	BTL -3	Applying
4. (a)	Find the Fourier Transform of $f(x)=\begin{cases}1-x^2\ , & if\ \ \lvert x\rvert \le 1\\ 0\ , & if\ \ \lvert x\rvert > 1\end{cases}$ Hence Show that $\int_0^\infty \left(\frac{\sin s - s\cos s}{s^3}\right)\cos\frac{s}{2}\,ds=\frac{3\pi}{16}$	BTL -1	Remembering
4.(b)	Find the Fourier Sine Transform of $f(x)=\begin{cases}x\ , & 0<x<1\\ 2-x\ , & 1<x<2\\ 0\ , & x>2\end{cases}$	BTL -2	Understanding

5.	Show that the Fourier transform of $f(x)=\begin{cases} a^2-x^2, & \lvert x\rvert < a \\ 0, & \lvert x\rvert > a > 0 \end{cases}$ is $2\sqrt{\frac{2}{\pi}}\left(\frac{\sin as - as\cos as}{s^3}\right)$.	BTL -3	Applying

	Hence deduce that $(i)\int_0^\infty \left(\frac{\sin t - t\cos t}{t^3}\right) dt = \frac{\pi}{4}$, $(ii)\int_0^\infty \left(\frac{\sin t - t\cos t}{t^3}\right)^2 dt = \frac{\pi}{15}$.		
6. (a)	Find the Fourier Transform of $f(x) = \frac{1}{\sqrt{\lvert x \rvert}}$.	BTL -2	Understanding
6.(b)	Using Parseval's Identity evaluate the following integrals. $(i)\int_0^\infty \frac{dx}{(a^2 + x^2)^2}$, $(ii)\int_0^\infty \frac{x^2 dx}{(a^2 + x^2)^2}$ where $a > 0$.	BTL -5	Evaluating
7. (a)	Find the Fourier sine transform of e^{-ax} (a>0). Hence find $F_s\left[xe^{-ax}\right]$ and $F_s\left[\frac{e^{-ax}}{x}\right]$ hence deduce the value of $\int_0^\infty \frac{\sin sx}{s} dx$	BTL -3	Applying
7. (b)	Evaluate $\int_0^\infty \frac{dx}{(x^2 + a^2)(x^2 + b^2)}$, using Fourier Transform	BTL -5	Evaluating
8. (a)	Find the function whose Fourier Sine Transform is $\frac{e^{-as}}{s}, a > 0$	BTL -1	Remembering
8.(b)	State and Prove Convolution Theorem on Fourier Transform	BTL -3	Applying
9. (a)	Find the Fourier Transform of $e^{-\lvert x \rvert}$ and hence find the Fourier Transform of $f(x) = e^{-\lvert x \rvert}\cos 2x$.	BTL -2	Understanding
9.(b)	Find the Fourier Cosine transform of e^{-4x}. Deduce that $\int_0^\infty \frac{\cos 2x}{x^2+16} dx = \frac{\pi}{8} e^{-8}$ and $\int_0^\infty \frac{x\sin 2x}{x^2+16} dx = \frac{\pi}{2} e^{-8}$.	BTL -2	Understanding
10.(a)	Find the Fourier Transform of $e^{-a\lvert x \rvert}$ and hence deduce that $(i)\int_0^\infty \frac{\cos xt}{a^2+t^2} dt = \frac{\pi}{2a} e^{-a\lvert x \rvert}$ $(ii)\ F\left[xe^{-a\lvert x \rvert}\right] = i\sqrt{\frac{2}{\pi}} \frac{2as}{(a^2+s^2)^2}$, hereF stands for Fourier Transform.	BTL -4	Analyzing
10.(b)	Using Fourier Sine Transform prove that $\int_0^\infty \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)} = \frac{\pi}{2(a+b)}$	BTL -3	Applying
11.(a)	Find the Fourier cosine & sine Transform of e^{-x}. Hence evaluate $(i)\int_0^\infty \frac{1}{(x^2+1)^2} dx$ and $(ii)\int_0^\infty \frac{x^2}{(x^2+1)^2} dx$.	BTL -2	Understanding
11.(b)	Find $F_s\left[xe^{-ax}\right]$ and $F_c\left[xe^{-ax}\right]$.	BTL -2	Understanding
12.(a)	Find the Fourier Sine Transform of the function $f(x) = \begin{cases} \sin x, & 0 \le x < a \\ 0, & x > a \end{cases}$	BTL -2	Understanding

12.(b)	Find the Fourier Cosine Transform of $f(x) = e^{-a^2x^2}$ and hence find the Fourier Cosine Transform of $e^{-\frac{x^2}{2}}$ and the Fourier Sine Transform of $xe^{-\frac{x^2}{2}}$.	BTL -2	Understanding
13.(a)	Find the Fourier sine transform of $\frac{s}{s^2+1}$ and Fourier cosine transform of $\frac{1}{s^2+1}$	BTL -2	Understanding
13.(b)	Using Parseval's Identity evaluate $\int_0^\infty \frac{dx}{(x^2+25)(x^2+9)}$.	BTL -5	Evaluating
14.(a)	Verify the Convolution Theorem for Fourier Transform if $f(x) = g(x) = e^{-x^2}$	BTL -5	Evaluating
14.(b)	Prove that $F_c[x\,f(x)] = \frac{d}{ds}[F_s\{f(x)\}]$ and $F_s[x\,f(x)] = -\frac{d}{ds}[F_c\{f(x)\}]$	BTL -3	Applying

UNIT -V Z - TRANSFORMS AND DIFFERENCE EQUATIONS

Z- Transforms - Elementary properties – Inverse Z - transform (using partial fraction and residues) – Initial and Final value theorem – Convolution theorem - Formation of difference equations – Solution of difference equations using Z - transform.

PART –A

Q.No.	Question	Bloom's Taxonomy Level	Domain
1.	Define Z – Transform of the sequence $\{f(n)\}$.	BTL -1	Remembering
2.	Find $Z(3^{n+2})$ and $Z\left[\cos^2\frac{n\pi}{2}\right]$	BTL -2	Understanding
3.	Find $Z\left[\frac{a^n}{n!}\right]$	BTL -2	Understanding
4.	Find $Z\left[\frac{1}{n!}\right]$	BTL -2	Understanding
5.	Find $Z\left[\frac{□1}{n(n+1)}\right]$	BTL -2	Remembering
6.	Define the unit step sequence .write its z-transform	BTL -1	Remembering
7.	State initial value theorem and final value theorem	BTL -1	Remembering
8.	Find $Z[n^2]$.	BTL -2	Understanding
9.	Find inverse Z transform of $\frac{z}{(z-1)(z-2)}$	BTL -2	Understanding
10.	Find $Z\left[\frac{□1}{(n+1)!}\right]$	BTL -2	Understanding

11.	Find $Z\left[e^{t}\sin 2t\right]$.	BTL -2	Understanding

12.	Prove that $Z\left[a^n f(n)\right] = f\left(\frac{z}{a}\right)$	BTL -5	Evaluating
13.	Prove that $Z\left[a^n\right] = \frac{z}{z-a}$	BTL -5	Evaluating
14.	Find $z^{-1}\left[\frac{z}{(z-1)^2}\right]$	BTL -2	Analyzing
15.	Find Z transform of $\frac{1}{n}$	BTL -2	Understanding
16.	Find $z^{-1}\left[\frac{z}{(z+1)^2}\right]$	BTL -2	Understanding
17.	Solve $y_{n+1} + 2y_n = 0$ given that $y(0)=2$	BTL -3	Applying
18.	State Convolution theorem in Z – Transforms	BTL -1	Remembering
19.	Form the difference equation by eliminating arbitrary constants from $y_n = A2^{n+1}$	BTL -6	Creating
20.	Prove that $Z[f(n+1)] = zF(z) - zf(0)$.	BTL -6	Creating
	PART-B		
1.(a)	Find the z transform of $f(n) = \frac{2n+3}{(n+1)(n+2)}$	BTL -2	Understanding
1. (b)	Find the $Z^{-1}\left(\frac{10z}{z^2-3z+2}\right)$	BTL -2	Understanding
2. (a)	Find the z-transform of $(n+1)^2$ and $\sin(3n+5)$	BTL -2	Understanding
2.(b)	Find the inverse Z – Transform of $\frac{z(z^2-z+2)}{(z+1)(z-1)^2}$.	BTL -2	Understanding
3. (a)	Find $(i) Z\left[r^n \cos n\theta\right]$, $(ii)\ Z\left[r^n \sin n\theta\right]$ $iii) Z(e^{-at}\cos bt)$	BTL-2	Understanding
3.(b)	Using convolution theorem find inverse Z transform of $\left[\frac{z^2}{(z-a)(z-b)}\right]$	BTL -3	Applying
4. (a)	Find the $Z^{-1}\left[\frac{z}{(z-1)(z+2)}\right]$	BTL -2	Understanding
4.(b)	Using convolution theorem find the inverse Z – Transform of $\frac{12z^2}{(3z-1)(4z+1)}$	BTL -3	Analyzing
5. (a)	Find inverse Z -Transform of $\frac{z^3}{(z-1)^2(z-2)}$ by the method of Partial fraction	BTL -2	Understanding
5.(b)	Using convolution theorem find inverse Z transform of $\frac{8z^2}{(2z-1)(4z+1)}$.	BTL -3	Applying
6. (a)	Using residue find $Z^{-1}\left[\frac{4z^3}{(2z-1)^2(z-1)}\right]$.	BTL -3	Applying
6.(b)	Using convolution theorem find $Z^{-1}\left[\frac{z^2}{(z-4)(z-5)}\right]$	BTL -3	Applying

7.(a)	Using Z transform Solve $y_{n+2} - 3y_{n+1} + 2y_n = 0$ given that y(0)=0,y(1)=1	BTL -3	Analyzing
7.(b)	If $f(z) = \frac{2z^2+3z+12}{(z-1)^4}$, Find the values of u_2 and u_3 by residue method.	BTL -2	Understanding
8.(a)	Using Z transform solve $y_{n+2} + 4y_{n+1} + 3y_n = 3^n$ given that y(0)=0,y(1)=1	BTL -3	Applying
8.(b)	Using the inversion method (Residue theorem)find the inverse Z transform of $U(z)=\left[\frac{z^2}{(z+2)(z+4)}\right]$	BTL -3	Applying
9.(a)	Using Residue method find $Z^{-1}\left(\frac{z}{z^2-2z+2}\right)$	BTL -3	Applying
9.(b)	Using convolution theorem evaluate $Z^{-1}\left[\frac{z^2}{(z-1)(z-3)}\right]$	BTL -3	Applying
10.(a)	Using convolution theorem evaluate $Z^{-1}\left[\frac{z^2}{(z-\frac{1}{2})(z-\frac{1}{4})}\right]$	BTL -3	Applying
10.(b)	Using Z transform solve $y_{n+2} - 3y_{n+1} - 10y_n = 0$ with y(0)=0,y(1)=1	BTL -3	Applying
11.(a)	Find the Z transform $\{a^n\}$ and $\{na^n\}$	BTL -3	Applying
11.(b)	Using Z transform solve $y_{n+2} - 7y_{n+1} + 12y_n = 2^n$ given that y(0)=0,y(1)=0	BTL -3	Applying
12.(a)	Form the difference equation $y(k+3) - 3y(k+1) + 2y(k) = 0$ with $y(0) = 4, y(1) = 0$ and $y(2) = 8$	BTL -6	Creating
12.(b)	State and prove final value theorem and their inverse transformation	BTL -3	Applying
13.(a)	Find the Z transform of $\{\frac{1}{n}\}$ and $\{\cos n\frac{\pi}{2}\}$	BTL -2	Understanding
13.(b)	Solve the difference equation $y_{n+3} - 3y_{n+1} + 2y_n = 0$ given that y(0)=4 ,y(1)=0,y(2)=8	BTL -3	Applying
14.(a)	Solve the equation using Z – Transform $y_{n+2} - 5y_{n+1} + 6y_n = 36$ given that y(0) = y(1) = 0	BTL -3	Applying
14.(b)	Find $Z^{-1}\left(\frac{z}{z^2+7z+10}\right)$ by convolution theorem.	BTL -2	Understanding