UNIT I -CLASSIFICATION OF SIGNALS AND SYSTEMS

PART A

1. What are the major classifications of signals?

Signals are classified as Continuous Time (CT) and Discrete Time(DT) signals.
Both CT and DT signals are further classified as
Deterministic and Random signals, Even and Odd signals, Energy and Power signals, Periodic and Aperiodic signals

2. With suitable examples distinguish a deterministic signal from a random signal. Define a random signal.(May 2013)

Deterministic signal: A signal which can be modeled (represented) by a mathematical equation.
Example: cosine signal

Random signal: A signal which cannot be modeled by a mathematical equation is called random signal.

Example: Speech Signal

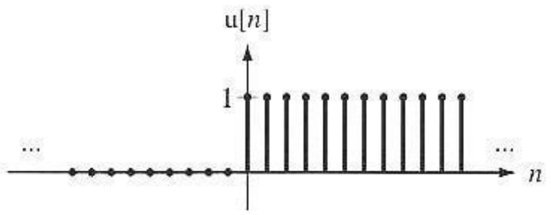
3. Define energy signal and power signal.(April 2015)

A signal x(t) is said to be energy signal if , Energy is finite i.e. $0 < E < \infty$ and average power is zero i.e. P=0 Where E = energy and P = Average power

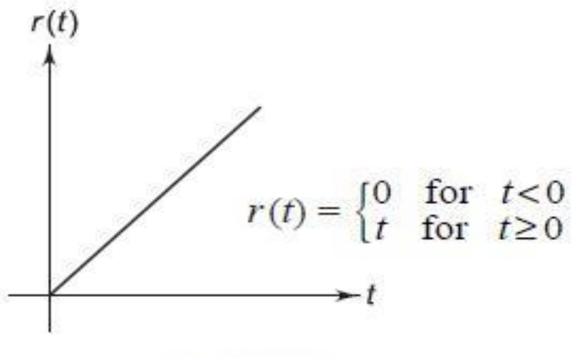
A signal x(t) is said to be power signal if power is finite i.e $0 < P < \infty$ and energy is infinite i.e. $E = \infty$ where E = energy and P = Average power

4. Give the mathematical and graphical representation of unit step sequence.

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



5. Give the mathematical and graphical representation of unit ramp signal.



Ramp function

6. What are periodic signals? Give example.

A signal $x(t)$ is said to be periodic if $x(t) = x(t+T)$ for all „t‟. The smallest value of „ T „ for which the condition is satisfied is called the fundamental period.Example: sinusoidal signals

7. What is odd signal? Give example.

A signal $x(t)$ is said to be odd signal if $x(t) = x(-t) = -x(t)$ .Example: $x(t) = A\sin\omega t$

8. State two properties of unit impulse function. (Dec2014)

1.δ(t) is the limit of graphs of area 1, the area under its graph is 1. δ(t)peak response at origin.

9. Define symmetric and anti symmetric signal.

Symmetric signal: It is a even signal,A signal $x(t)$ is said to be symmetric signal if $x(-t) = x(t)$.

Example: $x(t) = A\cos\omega t$

Anti symmetric signal:

A signal $x(t)$ is said to be anti symmetric signal if $x(t) = -x(-t)$. Example: $x(t) = A\sin\omega t$

10. Verify whether $x(t) = Ae^{-at}u(t)$, $a > 0$ is an energy signal or not.

$$x(t) = Ae^{-at}u(t),\ a > 0 : \ Energy = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt = \lim_{T\to\infty} \int_{0}^{T} |Ae^{-at}|^2 dt = \lim_{T\to\infty} \left[A^2 \frac{e^{-2at}}{-2a} \right]_0^T = \frac{A^2}{2a}\ Joules$$

$$power = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T} \left[A^2 \frac{e^{-2at}}{-2a} \right]_0^T = 0\,[Watt]$$

Energy is finite, Power is zero. The signal is energy signal

11. Show that the complex exponential signal $x(t) = e^{j\omega_0 t}$ is periodic and that the fundamental period is $\frac{2\pi}{\omega_0}$.

$$x(t) = e^{j\omega_0 t} : x(t+T) = e^{j\omega_0 (t+T)} = e^{j\omega_0 t} e^{j\omega_0 T}$$

$$x(t) = x(t+T) \quad when \quad e^{j\omega_0 T} = 1$$

$when \quad \omega_0 T = 2\pi m \quad where \quad m = integer \quad smallest\ value$ for $m = 1$, fundamental period = $2\pi/\omega_0$

12. Determine the power and RMS value of the signal $x(t) = e^{jat}\cos\omega_0 t$.

$$power = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |e^{jat} \cos \omega_0 t|^2 dt = \lim_{T\to\infty} \frac{1}{4T} \int_{-T}^{T} (1 + \cos 2\omega_0 t) dt \qquad since\ |e^{jat}| = 1$$

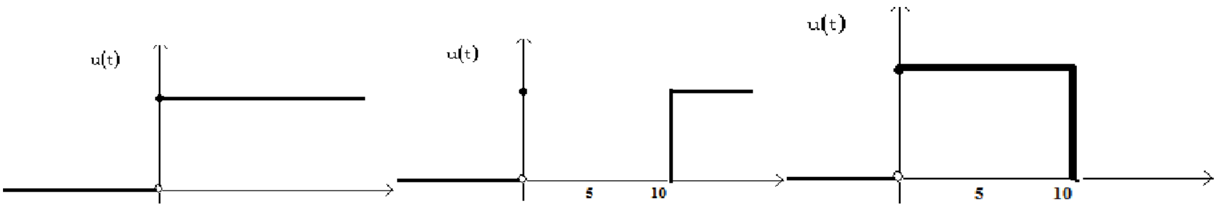
$$= \lim_{T\to\infty} \frac{1}{4T}(2T) = \frac{1}{2}\ watt \quad ; RMS\ value = \sqrt{\frac{1}{2}}$$

13.Find the average power of the signal $u(n) - u(n-N)$.

Average power of a DT signal x(n) is

$$Power = \lim_{N\to\infty} \frac{1}{(2N+1)} \sum_{n=-N}^{N} |x(n)|^2 = \lim_{N\to\infty} \frac{1}{(2N+1)} \sum_{n=0}^{N} (1)^2 = 0\ watt$$

14. Draw the following signal x (t) = [u (t) – u (t – 10)] .(Nov2014)



15. Give the formula for decomposing an arbitrary signal x(t) in to odd and even part.
CT signal:

$$x(t) = x_e(t) + x_o(t) : x_e(t) = even\ component \quad \& \quad x_o(t) = odd\ component$$

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)] \ ; x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

DT signal:

$$x(n) = x_e(n) + x_o(n) : x_e(n) = even\ component \quad \& \quad x_o(n) = odd\ component$$

$$x_e(n) = \frac{1}{2}[x(n) + x(-n)]; \ x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$

16. Give x[n] = {1,-4,3,1,5,2}. Represent x[n] interms of weighted shifted impulse functions.(May2014)

x[n] can be represented interms of its weighted shifted impulse functions as follows:

x[n]= [n] - 4 [n-1]+3 [n-2]+ [n-3]+5 [n-4]+2 [n-5].

17. Distinguish static system from dynamic system.

Static system:

Static system is a system with no memory or energy storage element.

Output of a static system at any specific time depends on the input at that particular time.

Dynamic system:

Dynamic systems have memory or energy storage elements.

Output of a dynamic system at any specific time depends on the inputs at that specific time and at other times.

18. Define a time invariant system.

A system is said to be time invariant if its input-output characteristics do not change with time.

Let $y(t) = \Gamma[x(t)]$; Γ *denotes some transformation (operation) on* $x(t)$;x(t)-input,y(t)- output

Let $y(t, T)$ denote the output due to delayed input $x(t-T)$ i.e, $y(t, T) = \Gamma[x(t-T)]$

let $y(t-T)$ be the out put delayed by T if $y(t-T) = y(t, T)$ then the system is time invariant

19. Define a continuous time LTI system./ Give the conditions for a system to be LTI system. (Dec2013) A continuous time system which posses two properties i) linearity (Obeys superposition principle) ii) Time invariance(Input –output characteristics do not vary with time) is a CT LTI system.

20. Determine whether the system described by the following input-output relationship is linear and causal y(t) = x(-t)

$y(t) = x(-t)$ $\rightarrow$ *input –output relationship* $y(t) = output$ & $x(t) = input$

Checking for linearity:

For an input $x_1(t)$, the output $y_1(t)$ is, $y_1(t) = x_1(-t)$

For an input $x_2(t)$, the output $y_2(t)$ is, $y_2(t) = x_2(-t)$

For an input $a\,x_1(t) + b\,x_2(t)$, the output $y_3(t)$ is, $y_3(t) = a\,x_1(-t) + b\,x_2(-t)$

$y_3(t) = a\,y_1(t) + b\,y_2(t)$ The system obeys superposition principle. Therefore the system is linear

Checking for causality:

For $t = -1$, $y(-1) = x(1)$ For negative values of time „t", the output depends on the future input.

Therefore the system is non-causal.

21. Determine whether the following signal is energy or power signal. And calculate its energy or power x(t)=e-^{2t}u(t).(Dec 2012)

$$Energy = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt = \lim_{T\to\infty} \int_{0}^{T} |e^{-2t}|^2 dt = \lim_{T\to\infty} \left[\frac{e^{-4t}}{-4}\right]_0^T = \frac{1}{4}\ Joules$$

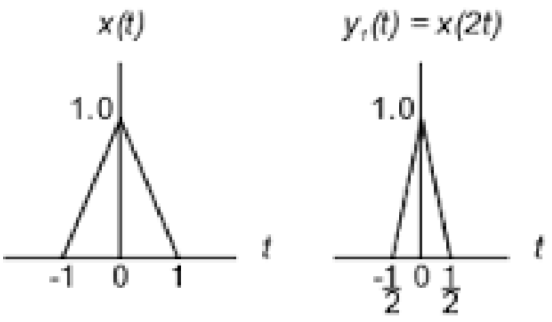
$$power = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T} \left[\frac{e^{-4t}}{-4}\right]_0^T = 0\,Watt$$

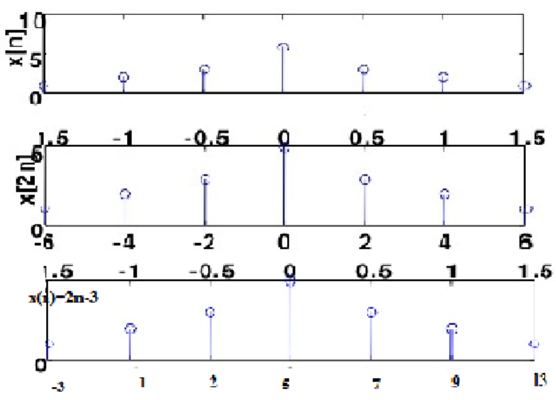
Energy is finite,Power is zero. The signal is energy signal

22. Check whether the following system is static (or) dynamic and also causal (or) non- causal:y(n)=x(2n) (Dec 2012)

For a given „n" the output depends on the future input. Therefore the system is non-causal.
The system is a dynamic system.

23. Sketch the following signal x(t)=2t and x(n)=x(2n-3)(May 2014)





24.Find the fundamental period of the given signal $x(n) = \sin\left(\frac{6\pi n}{7} + 1\right)$ **(May2012)**

$\omega_0 = \frac{6\pi}{7}$; $\omega_0 N = 2\pi k$; $N = \frac{k\,2\pi \times 7}{6\pi} = k \times \frac{14}{6}$ *where k , N are integer*

for some value of k, N takes the lowest possible integer value when $k = 6$, $N = 14$

x(n) is periodic with period N=14

25. Verify whether the system described by the equation is linear and time invariant.y(t)=x(t^2) Linearity:

$y(t) = x(t^2)$; $y(t) = T[x(t)] = x(t^2)$

For an input x1(t), $y_1(t) = T[x_1(t)] = x_1(t^2)$

For an input $x_2(t)$, $y_2(t) = T[x_2(t)] = x_2(t^2)$

Weighted sum of outputs is given by $ay_1(t) + by_2(t) = ax_1(t^2) + bx_2(t^2)$

Output due to weighted sum of inputs is $y_3(t) = T[ax_1(t) + bx_2(t)] = [ax_1(t^2) + bx_2(t^2)]$

Therefore, the system is linear.

Time invariance:

$y(t) = x(t^2)$, $y(t) = T[x(t)] = x(t^2)$

If the input is delayed by k units of time then the output is , $y(t, k) = T[x(t-k)] = x((t-k)^2)$

Output delayed by k units of time is, $y(t-k) = x(t^2 - k)$, $y(t, k) \neq y(t-k)$

Therefore, the system is time –variant.

26. **Give the relationship between unit impulse function $\delta(t)$, step function $u(t)$ and ramp function $r(t)$.(Nov 2015)**

$\delta(t) = \frac{d}{dt}u(t)$,$u(t) = \frac{d}{dt}r(t)$

27. How the impulse response of a discrete time system is useful in determining stability and causality?(April 2015)

For a causal system impulse response h(n)=0 for $n \leq 0$

For stable system system function poles located inside unit circle.

PART B

1. i) Sketch the signal x (t) = u (t) – u (t - 15). Determine the energy and power in the signal x(t).

   ii) Determine the energy and power in the signal $x(n) = \left(\frac{1}{\ }\right)^n u(n)$

2. i) How are signals classified.

   ii) Determine whether the following signal is periodic. If periodic, determine the fundamental period: $x(t) = 3\cos(t) + 4\cos(t/3)$ **(Dec 2012)**

   iii) Give the equation and draw the waveforms of discrete time real and complex exponential signals.

3. i) Define an energy and power signal (4)

   ii) Determine whether the following signals are energy or power and calculate their energy or power.(12). **(May 2013)**

   (i) $x(n) = \left[\frac{1}{2}\right]^n u(n)$ (ii) $x(t) = rect\left[\frac{t}{T_o}\right]$ (iii) $x(t) = \cos^2(\Omega_0 t)$

4. i) Define unit step, Ramp, Pulse, Impulse and exponential signals. Obtain the relationship between the unit step function and unit ramp function. (10)

   ii) Find fundamental period $x(n) = \cos\left(\frac{\pi n}{2}\right) - \sin\left(\frac{\pi n}{8}\right)\cos\left(\frac{\pi n}{4} + \frac{\pi}{3}\right)$ **(May 2013)**

5. Determine whether the discrete time system y(n)=x(n)cos(ωn) is (i) memoryless (ii) Stable (iii) causal (iv) linear (v)time invariant.**(Dec 2013)**

6. i) Determine whether the signal x(t)=sin20πt+sin5πt is periodic and if it is periodic find the fundamental period(5) **(Dec 2013)**

   ii) Discuss various forms of real & complex exponential signals with graphical representations (6) **(Dec 2013)**

   iii) State the precedence rule for combined time scaling and time shifting operation..(5)

7. Check whether the system is linear, causal, time invariant and or stable **(Dec2014)**

   i) y(n) = x(n) - x(n-1) ii) $y(t) = \frac{d}{dt}x(t)$

8. Check whether the following signals are periodic/aperiodic signals.**(Dec2014)**

   $x(t) = \cos 2t + \sin\left(\frac{t}{5}\right)$ $x(n) = 3 + \cos\left(\frac{\pi n}{2}\right) + \cos 2n$

9. (i) Given $x(t) = \frac{1}{6}(t+2)$ $-2 \leq t \leq 4$ $otherwise\ 0$ Sketch (1) x(t) (2) x(t+1) (3) x(2t) (4)x(t/2).

   (ii) Determine whether the discrete time sequence $x[n] = \sin\left(\frac{3\pi}{7}n + \frac{\pi}{4}\right) + \cos\left(\frac{\pi}{3}\right)$ is periodic or not. **(May2014)**

10. Check the following systems are linear, stable

    (i) $y(t) = e^{x(t)}$ (ii)y(n)=x(n-1). **(May2014)**

11. Given x(n)=[1,4,3,-1,2]. Plot the following signals **(Dec2015)**

$i)\ x(n-1)\quad ii)\ x\left(\frac{-n}{2}\right)\qquad iii)\ x(-2n+1)\quad iv)\ x\left(\frac{-n}{2}+2\right)$

12. Given the input-output relationship of a continuous time system y(t)=tx(-t). Determine whether the system is is linear, causal ,time invariant and stable **(Dec2015)**
13. i) Give an account for classification of signals in detail (10) **(May 2015)**
    ii) Sketch the following signals a) u(t-2)+u(t-4) b) (t-4) [u(t-2) – u(t-4)] (6)
14. i) Check if x(t)= 4cos(3πt+ π/4) + 2 cos (4πt) is periodic.(6) **(May 2015)**
    ii) For the system y(n) = log[x(n)], Check for linearity, causality, time invariance and stability.(10)

UNIT II-ANALYSIS OF CONTINUOUS TIME SIGNALS

PART-A

1. **State the conditions for convergence of Fourier series.(May2014)**

   The Fourier Series exists only when the function x(t) satisfies the following three conditions:
   a) the function x(t) have only a finite number of maxima and minima
   b) the function x(t) have a finite number of discontinuities
   c) x(t) is absolutely integrable.

   i.e, $\int_0^T |x(t)|dt < \infty$

2. **State Dirichlet's conditions for Fourier Transform.(Dec2013/Nov 15)**

   The Fourier transform does not exist for all aperiodic functions. The conditions for x(t) to have Fourier transform are:
   a) x(t) is absolutely integrable over (– ∞, ∞)

   ie., $\int_{-\infty}^{\infty} |x(t)\ dt| < \infty$

   b) x(t) has finite number of discontinuities and a finite number of maxima and minima in every finite time interval.

3. **Give the Fourier transform and Inverse Fourier transform pair equation.**

   $X(j\Omega) = F[x(t)] \qquad and\ x(t) = F^{-1}[X(j\Omega)]$

   $X(j\Omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\Omega t}\, dt \qquad for\ all\ \Omega$

   $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\Omega)\, e^{j\Omega t}\, d\Omega \qquad for\ all\ t$

4. **State modulation property of Fourier transform.**

   $F[x(t)] = X(j\Omega) \quad ; F\left[e^{j\Omega_0 t} x(t)\right] = X[j(\Omega - \Omega_0)]$

   Similarly, $F\left[e^{-j\Omega_0 t} x(t)\right] = X[j(\Omega + \Omega_0)]$

5. **State convolution(time) property of Fourier transform.**

   $F[x(t)] = X(j\Omega)\ ; F[h(t)] = H(j\Omega)$

   $F[x(t) * h(t)] = X(j\Omega)H(j\Omega)$

6. **What is the Fourier transform of $x(t) = e^{-at}u(t)$?**

   $X(j\Omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\Omega t}\, dt = \int_0^{\infty} e^{-at} e^{-j\Omega t} dt = \left[-\frac{e^{-(a+j\Omega)t}}{(a+j\Omega)}\right]_0^{\infty} = \frac{1}{(j\Omega + a)}$

7. **What is the Inverse Fourier transform of** $X(j\Omega) = \frac{1}{(a + j\Omega)^2}$ **?**

   $x(t) = t\, e^{-at}\, u(t)$

8. **Find the Fourier transform of the impulse signal.**

   $x(t) = \delta(t) \qquad ; X(j\Omega) = \int_{-\infty}^{\infty} \delta(t)\, e^{-j\Omega t}\, dt = 1$

9. **Determine Laplace transform of x(t)=e^{-at}sin(Ωt)u(t).**

$$L[\sin(\Omega t)u(t)] = \frac{\Omega}{s^2 + \Omega^2}$$

$$X(S) = L\left[e^{-at}\sin(\Omega t)u(t)\right] = \frac{\Omega}{(s+a)^2 + \Omega^2}$$

10. **Find the ROC of the Laplace transform of x(t)=u(t).(Nov2014)**
The ROC of u(t) is entirely outside of the Unit circle. Since X(S)=1/(s+1)

11. **What is the inverse Laplace transform of** $\frac{1}{(s+2)}$; $\text{Re}\{s\} < -2$

Inverse Laplace transform of $\frac{1}{(s+2)}$; $\text{Re}\{s\} < -2$ is, $-e^{-2t}u(-t)$

12. What is the inverse Laplace transform of $\frac{1}{(s+1)}$; $\text{Re}\{s\} > -1$

Inverse Laplace transform of $\frac{1}{(s+1)}$; $\text{Re}\{s\} > -1$ is, $e^{-t}u(t)$

13. Define ROC of Laplace transform. (Dec 2012)
Laplace transform of x(t) is given by the following formula

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt$$

The range of values of „s" for which the integral in the equation converges is referred to as the region of convergence(ROC).

14. What is the Laplace transform of (i) u(t) (ii) t u(t) ? Also specify the ROC.

(i) Laplace transform of u(t) = $\int_0^{\infty} e^{-st}\,dt = \left[-\frac{e^{-st}}{s}\right]_0^{\infty} = \frac{1}{s}$ $\quad ROC:\ \text{Re}\{s\} > 0$

(ii) Laplace transform of tu(t) = $\int_0^{\infty} te^{-st}\,dt = \left[-\frac{t\,e^{-st}}{s} - \frac{e^{-st}}{s^2}\right]_0^{\infty}$

$$= \frac{1}{s^2} \qquad ROC:\ \text{Re}\{s\} > 0$$

15. Determine the Laplace transform and ROC for the signal $y(t) = -e^{at}u(-t)$ **.**

$$Y(s) = \int_{-\infty}^{\infty} -e^{-at}u(-t)\,e^{-st}\,dt = -\int_{-\infty}^{0} e^{-(s-a)t}\,dt = \frac{1}{(s-a)} \qquad ROC:\ \text{Re}\{s\} < a$$

16. Determine the Laplace transform and ROC of x(t)=u(t-5).(May2012)

$$f(t) \overset{LT}{\longleftrightarrow} F(s) \quad ; f(t-a) \overset{LT}{\longleftrightarrow} e^{-as}F(s)$$

Laplace Transform of $u(t-5) = e^{-5s}L[u(t)] = e^{-5s} \times \frac{1}{s}$

$$L[u(t-5)] = \frac{e^{-5s}}{s} \qquad ROC:\ \text{Re}\{s\} > 0$$

17. What is the unilateral Laplace transform of $\frac{d}{dt}x(t)$ **?**

$$\frac{dx(t)}{dt} \overset{unilateral\ LT}{\longleftrightarrow} s\,X(s) - x(0^-)$$

18. **State the convolution (in time) property of Laplace transform.** $x(t) \overset{LT}{\longleftrightarrow} X(s)$; $h(t) \overset{LT}{\longleftrightarrow} H(s)$

$$x(t) * h(t) \overset{LT}{\longleftrightarrow} X(s)H(s)$$

19. State the time scaling property of Laplace transform.(May2013)

$$x(t) \overset{LT}{\longleftrightarrow} X(s) \qquad ROC: R$$

$$x(at) \overset{LT}{\longleftrightarrow} \frac{1}{|a|} X\left(\frac{s}{a}\right) \qquad ROC: \frac{R}{|a|}$$

20. State initial value theorem and final value theorem of Laplace transform.

$L[x(t)] = X(s)$

Initial Value theorem: $\lim_{t \to 0} x(t) = \lim_{s \to \infty} s\,X(s)$

Final Value theorem: $\lim_{t \to \infty} x(t) = \lim_{s \to 0} s\,X(s)$

21. Find the initial and final values for $X(s) = \dfrac{(s+5)}{(s^2+3s+2)}$.

$$\lim_{t \to 0} x(t) = \lim_{s \to \infty} s\,X(s) = \lim_{s \to \infty} \frac{s^2+5s}{s^2+3s+2} = \lim_{s \to \infty} \frac{1+\frac{5}{s}}{1+\frac{3}{s}+\frac{2}{s^2}} = 1$$

$$\lim_{t \to \infty} x(t) = \lim_{s \to 0} s\,X(s) = \lim_{s \to 0} \frac{s^2+5s}{s^2+3s+2} = 0$$

22. Give synthesis and analysis equations of CT Fourier transform.(Dec 2012)

$X(j\Omega) = F[x(t)]$ *and* $x(t) = F^{-1}[X(j\Omega)]$

$X(j\Omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\Omega t}\, dt$ for all $\Omega \to$ analysis equation

$x(t) = \dfrac{1}{2\pi}\int_{-\infty}^{\infty} X(j\Omega) e^{j\Omega t}\, d\Omega$ for all $t \to$ synthesis equation,

23. Determine the Fourier series coefficients for the signal $x(t) = \cos(\pi t)$**.(May 2012)**

$\Omega_0 = \pi$

$x(t) = \sum_{n=-\infty}^{\infty} C_n\, e^{jn\Omega_0 t}$ $\Rightarrow$ *Exponential Fourier Series*

From the given equation, $x(t) = \cos(\pi t) = \dfrac{1}{2}\left[e^{j(\pi)t} + e^{-j(\pi)t}\right]$

$$= \frac{1}{2}\left[e^{j\pi t}\right] + \frac{1}{2}\left[e^{-j\pi t}\right] = C_{-1} e^{-j\Omega_0 t} + C_1 e^{j\Omega_0 t}$$

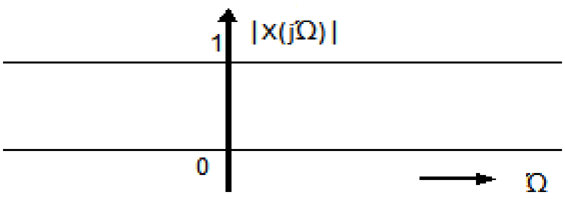
where $C_{-1} = \dfrac{1}{2}$ & $C_1 = \dfrac{1}{2}$

24. What is the fourier transform of a DC signal of amplitude 1? (May2013)

$x(t) = 1$

$X(j\Omega) = \int_{-\infty}^{\infty} 1.\, e^{-j\Omega t}\, dt = 1$

$|X(j\Omega)| = 1$ *for all* Ω $\quad$ $angle(X(j\Omega)) = 0$ *for all* Ω



25. State any two properties of ROC of Laplace transform X(S) of a signal x(t).(May2014)

i)If x(t) is absolutely integrable and of finite duration, then the ROC is the entire s-plane (the Laplace transform integral is finite, i.e., X(S) exists, for any s.

ii)The ROC of X(S) consists of strips parallel to the jω-axis in the s-plane.

iii)If x(t) is right sided and Re(s)=σ0 is in the ROC, then any s to the right of σ0 (i.e., Re(s)> σ0) is also in the ROC, i.e., ROC is a right sided half plane.

iv)If x(t) is left sided and Re(S)= σ0 is in the ROC, then any s to the left of σ0 (i.e., Re(S)< σ0) is also in the ROC, i.e., ROC is a left sided half plane.

26.What is the relation between Laplace and Fourier transform? (Nov 15)

$$L\,(x(t)) = X\,(s) = \int_{-\infty}^{\infty} x(t)e^{-s\,t}\,dt \qquad \textit{Laplace transform}$$

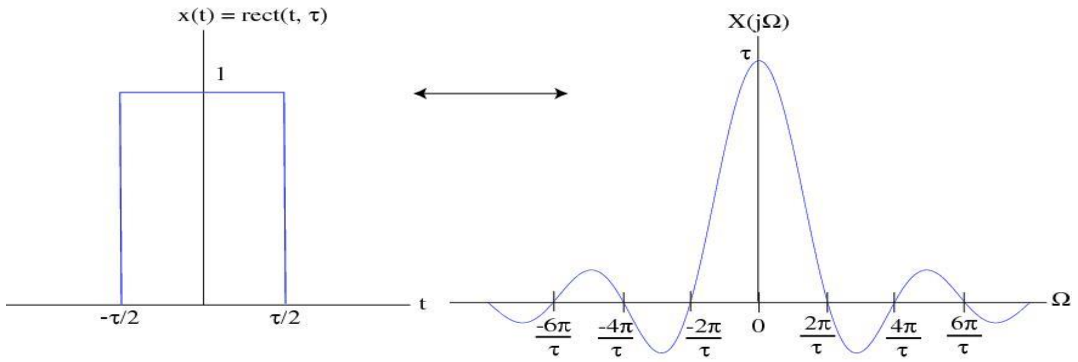
$$F\,(x(t)) = X\,(j\Omega) = \int_{-\infty}^{\infty} x(t)e^{-j\Omega t}\,dt \textit{ Fourier transform} \quad S = j\Omega$$

27. **What is the Laplace transform and Fourier Transform of δ(t) (Apr15) ?**

$$L\,(\,x(t\,)) = X\,(S\,) = \int_{-\infty}^{\infty} \delta\,(t\,)e^{-\,S\,t}\,dt \;=\; 1$$

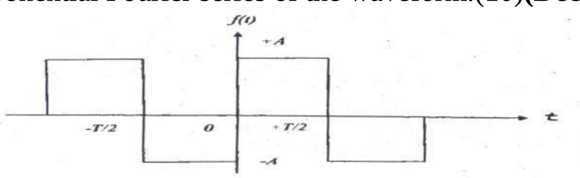
$$F\;\;(\,x(t\,)) = X\,(\,j\Omega) = \int_{-\infty}^{\infty} x(t\,)e^{-\,j\Omega t}\,dt\;=1$$

28.Draw the Spectrum of CT Rectangular Pulse?(Apr15)

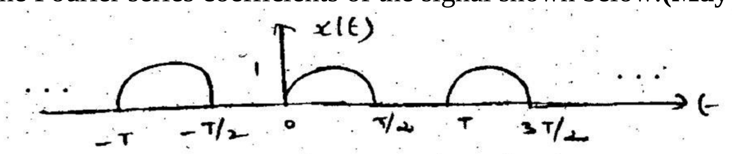


PART B

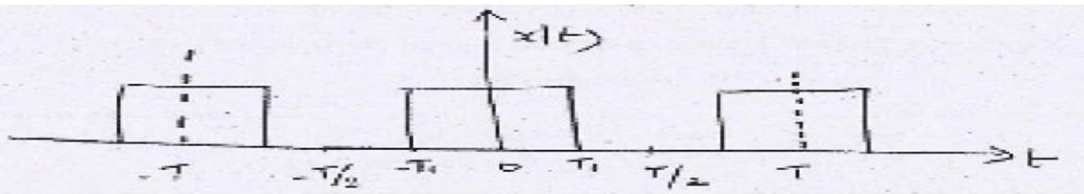
1. i) State Dirichlet"s conditions. Also state its importance.
   ii) obtain the trigonometric Fourier series of the half wave rectified sine wave signal of period T and amplitude A. **(Dec 2012)**
2. i) Determine the Fourier transform for double exponential pulse whose function is given by $x(t) = e^{-2|t|}$ .Also draw its amplitude and phase spectra.**(Dec 2012)**
   ii)Obtain the inverse Laplace transform of the function
   $X\,(S\,) = \dfrac{1}{S^2 + 3S + 2}$, $ROC : -2 < \text{Re}\{S\} < -1$
3. i) Compute the Laplace transform of x (t)= $e^{b|t|}$ for the cases of b<0 and b>0 (10)**(May2013)**
   ii) State and prove Parseval"s theorem of Fourier Transform.(6)
4. i) Determine the fourier transform representation of the half wave rectifier output.(8)
   ii) Write the properties of ROC of Laplace transform. (8) **(May 2013)**
5. i) Find the exponential Fourier series of the waveform.(10)**(Dec 2013)**



   ii) Find the Fourier transform of the signal x (t)= $e^{a|t|}$ (6)
6. i) Find the Laplace transform of the signal f(t)= x (t)=e^{-at} sinωt.(8)**(Dec 2013)**
   ii) Find the Inverse Fourier transform of the rectangular spectrum given by
   (8) $x(\,j\omega) = 1 \quad -W \le \omega \le W$
   $= 0 \quad |\omega| \ge W$
7. Find the Fourier series coefficients of the signal shown below.**(May 2014)**



8. Find the inverse Laplace transform of 1/{(s+3)(s+5)}for the ROCs (i) -5<Re{s}<3 (ii) Re{s}>3
9. Find the Fourier series coefficients of the following signal. **(Dec 2014)**



10.Find the spectrum of x(t)=e-2|t|.Plot the spectrum of the signal.**(Dec2014)**

11. State and prove any four properties of Fourier Transform **(Dec2015)**

12.Find the Laplace transform and its associated ROC for the signal $x(t) = te^{-2|t|}$ **(Dec2015)**

13. i) Determine the fourier series expansion for a periodic ramp signal with unit amplitude and a period T (10)

ii) Find the Fourier transform of $x(t) = te^{-at} u(t)$ (6) **(May 2015)**

14.i) If x(t) <-> X(ω), then using time shifting property show that x(t+T) +x(t-T) <-> 2 X(ω)cosωT (6)

ii) Find the inverse Laplace transform of $X(S) = \dfrac{8S+10}{(S+1)(S+2)^3}$ (10) **(May 2015)**

<u>UNIT III-LINEAR TIME INVARIANT CONTINUOUS TIME SYSTEMS</u>

<u>PART A</u>

1. What is the overall impulse response h(t) when two systems with impulse responses $h_1(t)$ and $h_2(t)$ are in series?

Over all impulse response h(t) of two LTI systems with impulse responses $h_1(t)$ and $h_2(t)$ connected in cascade(series) is the convolution of the individual impulse responses.

Overall impulse response is $h(t) = h_1(t) * h_2(t)$

2. What is the overall impulse response h(t) when two systems with impulse responses $h_1(t)$ and $h_2(t)$ are in parallel?

Overall impulse response h(t) = sum of the individual impulse responses.

$$h(t) = h_1(t) + h_2(t)$$

3. What is meant by impulse response?

It is the response of the system to unit impulse signal $\delta(t)$ .It is denoted by h(t) and can be determined by taking inverse Laplace transform of the system transfer function.

4. A LTI system is characterized by the following differential equation $\dfrac{dy(t)}{dt} + ay(t) = x(t)$ **. Find the impulse response of the system.**

Assuming zero initial conditions, taking Laplace transform,

$$sY(s) + a\,Y(s) = X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{(s+a)} = \textit{system transfer function}$$

$$h(t) = \textit{impulse response} = L^{-1}\left[H(s)\right] = e^{-a\,t}\,u(t)$$

5. What are the drawbacks of representing a system using its transfer function?

a) The transfer function describes only the zero state response of a system.
b) It describes only the relationship between the input and output of a system, but does not provide any information regarding the internal state of a system.
c) It is limited to single input and single output systems.
d) It is applicable only for LTI systems.

6. Define system transfer function.

The transfer function of a system is defined as the Laplace transform of the impulse response h(t).

$$H(s) = \frac{Y(s)}{X(s)} = \frac{\textit{Laplace transform of the output}}{\textit{Laplace transform of the input}} \qquad \textit{with all initial conditions zero}$$

7.Determine the frequency response of the system having impulse response $h(t) = \delta(t) - 2e^{-2t}u(t)$ **.**

Frequency response = Fourier transform of impulse response.

$$F[h(t)] = H(j\Omega) = 1 - \frac{2}{(j\Omega + 2)} = \frac{j\Omega}{(j\Omega + 2)}$$

8. Check the causality of the system with impulse response $h(t)=e^{-t}u(t)$(Dec 2012)

At any given time, out put of the given system does not depend on future input . The system is causal.

9. Define a CT Linear system.

A continuous time system which obeys superposition principle.

10. Find the transfer function of the LTI system described by the differential equation

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = 2\frac{dx(t)}{dt} - 3x(t)$$

Taking Laplace transform of the equation , assuming zero initial conditions,

$$S^2Y(S) + 3SY(S) + 2Y(s) = 2SX(S) - 3X(S)$$

$$Transfer\ \ function = \frac{Y(s)}{X(s)} = H(s) = \frac{2s-3}{(s^2+3s+2)}$$

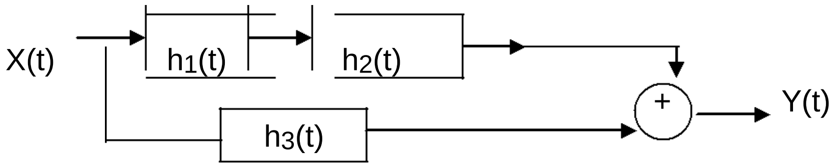
11.The impulse response of the LTI CT system is given as $h(t) = e^{-3t}u(t)$. Determine the system transfer function.

$$h(t) = e^{-3t}u(t);\ H(s) = L[h(t)] = \frac{1}{(s+3)} = transfer\ \ function$$

12. Let the impulse response of a LTI system be $h(t) = \delta(t-a)$. What is the output of this system in response to an input $x(t)$?

$$h(t) = \delta(t-a) : H(s) = e^{-as} = \frac{Y(s)}{X(s)} : Y(s) = H(s)X(s) = e^{-as}X(s) : y(t) = L^{-1}[Y(s)] = x(t-a)$$ **13**

13.Find the overall impulse response h(t) of the system shown.



Overall impulse response $= h(t) = [h_1(t) * h_2(t)] + h_3(t)$

14. What is the overall impulse response h(t) when two systems with impulse responses $h_1(t)= \delta(t)$ and $h_2(t) = e^{-t}u(t)$ are in series?

Over all impulse response h(t) of two LTI systems with impulse responses $h_1(t)$ and $h_2(t)$ connected in cascade(series) is the convolution of the individual impulse responses.

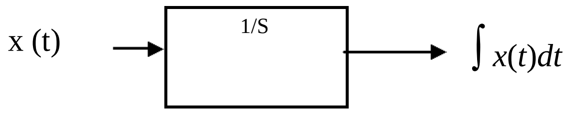
Overall impulse response is $h(t) = h_1(t) * h_2(t)$

$$in\ \ 's'\ domain\qquad H(s) = H_1(s)H_2(s)$$

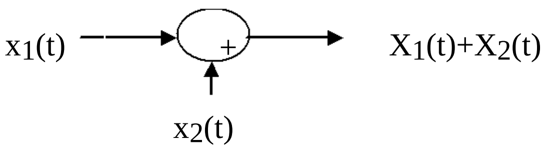
$$H_1(s) = L[\delta(t)] = 1 \qquad \&\ H_2(s) = \frac{1}{(s+1)} :\ H(s) = 1\times\frac{1}{(s+1)} = \frac{1}{(s+1)} : h(t) = L^{-1}[H(s)] = e^{-t}u(t)$$

15. List and draw the basic elements for the block diagram representation of the CT system. (Dec 2012)

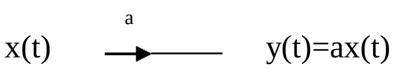
Integrator:



Summer:



Scalar multiplier:



16. Check the stability of the CT system whose impulse response is $h(t) = e^{-3t}u(t)$.

$$h(t) = e^{-3t}u(t);\ H(s) = L[h(t)] = \frac{1}{(s+3)} = transfer\ \ function$$

Pole is at s= - 3 which is on the left half of S-plane. Therefore the system is stable.

17.Compare the hardware requirements of Direct form I and Direct form II realization.

Direct form II structures require lesser number of integrators.

18. Define the convolution integral.(May2013,Apr 2015)

$$y(t) = h(\tau) * x(\tau);\ y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau;\ where,\ h(t) = impulse\ response,\ \ x(t) = input$$

19. What is the condition for a LTI system to be stable?(May2013)

The poles of the LTI system should be on the left half of S-plane.

20.What are the three elementary operations in block diagram representation of CT system.(Dec2013)

(i) Summing,(ii) Scalar multiplication,(iii) Integration.

21. Check whether the system is stable $H(S) = \dfrac{1}{S-2}$ **transfer function(Dec2013)**

Pole is at s= 2 which is on the right half of S-plane. Therefore the system is unstable.

22.State the necessary and sufficient condition for an LTI continuous time system to be causal. (May2014)

An LTI continuous time system is causal if and only if its impulse response is zero for negative values of t.

23.Find the differential equation relating the input and output of a CT system represented by (May2014).

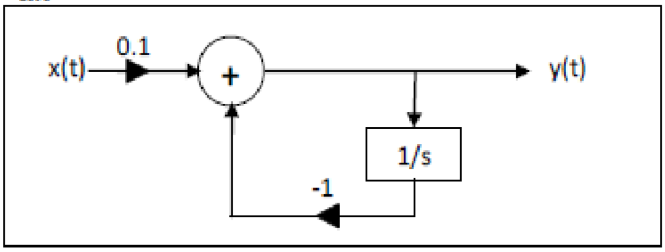
$$H(j\Omega) = \frac{Y(j\Omega)}{X(j\Omega)} = \frac{4}{(j\Omega)^2 + 8j\Omega + 4}$$

On cross multiplying, By taking inverse Fourier transform corresponding differential equation

$$\frac{d^2y(t)}{dt^2} + 8\frac{dy(t)}{dt} + 4y(t) = 4.x(t)$$

24.Draw the block diagram of the LTI system described by(Dec2014)

$$\frac{dy(t)}{dx} + y(t = 0.1x(t)$$



25.Find y(n)=x(n-1)*δ(n+2).(Dec2014)

y(n) = x(n-1+2)

= x(n+1)

26. .Given the differential equation representation of a system $\dfrac{d^2y(t)}{dt^2} + 2\dfrac{dy(t)}{dt} - 3y(t) = 2.x(t)$

Find the frequency response H(jΩ) (Dec 2015),

$$H(j\Omega) = \frac{Y(j\Omega)}{X(j\Omega)} = \frac{2}{((j\Omega)^2 + 2j\Omega - 3)}$$

PART B

1. i) What is impulse response? Show that the response of an LTI system is convolution integral of its impulse response with input signal.**(Dec 2012)**

ii) Obtain the convolution of the following two signals: $x(t) = e^{2t}u(-t)$, $h(t) = u(t-3)$ 2. The input and output y(t) for a system satisfy the differential equation

$$\frac{d^2y(t)}{dt} + 3\frac{dy(t)}{dt} + 2y(t) = x(t)$$

i) Compute the transfer function and impulse response.

ii) Draw direct form, cascade form and parallel form representations.**(Dec 2012)**

3. Determine the impulse response h(t) of the system given by the differential equation**(May 2013)**

$$\frac{d^2y(t)}{dtd(t)dt^2} + 5\frac{dy(t)}{} + 4y(t) = dx(t)$$

4. The system produces the output y(t) = e^{-t}u(t) for an input x(t) = e^{-2t} u(t). Determine (i) frequency response (ii)magnitude and phase of the response (iii) the impulse response **(May 2013)**

5. i) Define convolution Integral and derive its equation.(8)

ii) A stable LTI system is characterized by the differential equation **(Dec 2013)**

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = \frac{dx(t)}{d(t)} + 2x(t)$$

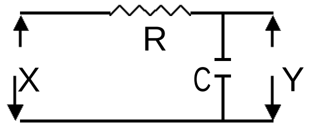
Find the frequency response and impulse response using Fourier transform.(8)

6. i) Draw the direct form , cascade form and parallel form of a system with system function **(Dec 2013)**

$$H(s) = \frac{1}{(s+1)(s+2)}$$

7. Determine the output response of RC Low pass network shown in figure due to input **(Dec2014)**

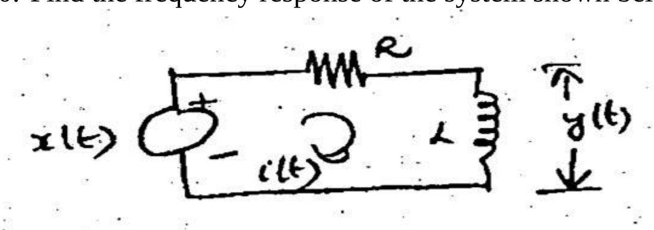
$x(t) = te^{-\frac{t}{RC}}$ by convolution.



8. An LTI system is represented by $\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 4y(t) = x(t)$ initial condition y(0-)=0; y'(0-)=1;Find the output of the system, when the input is $x(t) = e^{-t}u(t)$ **(Dec2014)**

9. Using convolution integral, determine the response of a CTLTI system y(t) given input $x(t) = e^{-\alpha t}u(t)$ and impulse response h(t) = $e^{-\beta t}$ u(t),|α|<1,|β|<1.**(May 2014)**

10. Find the frequency response of the system shown below :**(May 2014)**



11.Convolve the following signals : x(t) = e^{-2t}u(t-2) , h(t) = e^{-3t}u(t) **(Dec2015)**

12. The input-output of a causal LTI system are related by the differential equation

$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2x(t)$ .Use Fourier transform

i) Find the impulse response h(t)

ii) Find the response y(t) of this system if x(t) = u(t) **(Dec2015)**

13.i. Solve the differential equation (D^2+ 3D+2)y(t) = Dx(t) using the input x(t) = 10 e^{-3t} and with initial condition y(0^+) =2 and y·(0^+)=3 (10) **(May 2015)**

ii). Draw the block diagram representation for H(s) = (4s+28) / (s^2+6s +5) (6)

14.i) For a LTI system with H(s) = (s+5) / (s^2+ 4s +3)find the differential equation . Find the system output y(t) to the output x(t) = e^{-2t}u(t) (10)

ii) Using graphical method convolve x(t) = x(t) = e^{-2t}u(t) with h(t) =u(t+2) (6) **(May 2015)**

UNIT IV-ANALYSIS OF DISCRETE TIME SIGNALS

PARTA

1.State sampling theorem.(Nov 15)

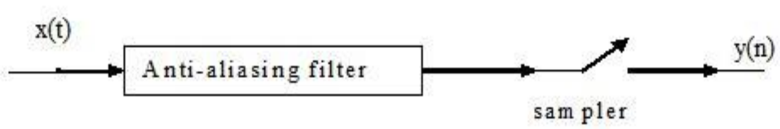
A continuous time (CT) can be completely represented in its samples and recovered back if the sampling frequency $F_s \geq 2F_m$.

$F_s = sampling\ \ frequency$

$F_m = highest\ \ frequency\ component\ present\ in\ the\ signal$

2. What is an antialiasing filter?(May2014)

A filter that is used to reject high frequency signals before it is sampled to reduce the aliasing is called an anti aliasing filter.



3. State any four properties of ROC for the z-transform.

If x(n) is a causal finite sequence then the ROC is the entire z-plane except at z = 0.
If x(n) is a anti causal sequence of finite duration then the ROC is the entire z-plane except at z = ∞.
ROC cannot contain any poles.
If x(n) is a finite duration two sided sequence the ROC is entire z-plane except at z = 0 and z = ∞.

4. Define Nyquist rate.(May 2012,)

A continuous time (CT) can be completely represented in its samples and recovered back if the sampling $F_s \geq 2F_m$ .F s sampling frequency, F m Maximum Frequency. The limiting sampling rate $F_s = 2F_m$ is called as Nyquist sampling rate.

5. State the time reversal property of z - transform.

$x(n) \xleftrightarrow{z} X(z) \quad ROC: R_1 < |z| < R_2$

$$x(-n) \xleftrightarrow{z} X(z^{-1}) \qquad ROC: \frac{1}{R_2} < |z| < \frac{1}{R_1}$$

6. For the analog signal(t)=3cos(50πt)+10sin(300πt)-cos(100πt), What is the minimum sampling rate required to avoid aliasing?

$$\cos(50\pi t) \Rightarrow \quad \Omega_1 = 50\pi \qquad F_1 = \frac{50\pi}{2\pi} = 25\ Hz$$

$$\sin(300\pi t) \Rightarrow \Omega_2 = 300\pi \qquad F_2 = \frac{300\pi}{2\pi} = 150\ Hz$$

$$\cos(100\pi t) \Rightarrow \Omega_3 = 100\pi \qquad F_3 = \frac{100\pi}{2\pi} = 50Hz$$

Highest frequency = 150 Hz = Fm
Minimum sampling rate = Nyquist rate = 2Fm = 300 Hz

7.What is the Z transform and ROC for the signal $x[n] = \delta(n-k) \quad , k > 0$

Using time shifting property of z-transform,

$$X(z) = z^{-k} \quad ROC: entire\ z-plane \quad except\ z = 0$$

8.Find the Z-transform of the DT sequence (Nov 15)

$$x(n) = \{1, -1, \underset{\uparrow}{2}, 3, 4\}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} = x(-2)z^2 + x(-1)z + x(0)z^0 + x(1)z^{-1} + x(2)z^{-2}$$

$$= \quad 1z^2 - z + 2z^0 + 3z^{-1} + 4z^{-2}$$

9. Find the Z transform of $x[n] = u[n-2] * \left(\frac{2}{3}\right)^n u[n]$

$$u[n-2] \xleftrightarrow{z} \frac{z^{-2}}{1-z^{-1}} \;;\; \left(\frac{2}{3}\right)^n u(n) \xleftrightarrow{z} \frac{1}{1-\frac{2}{3}z^{-1}} \;;\; X(z) = \frac{z^{-2}}{1-z^{-1}} \times \frac{1}{\left(1-\frac{2}{3}z^{-1}\right)} = \frac{1}{(z-1)z\left(z-\frac{2}{3}\right)}$$

10. Write the analysis and synthesis equation of DTFT.(Dec 2012)

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} \quad \text{->Analysis Equation}$$

$$x(n) = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega \quad \text{->Synthesis equation}$$

Note: also referred to as DTFT pair equation

11. State and prove time shifting property of DTFT.(May 2012)

$$x(n) \xleftrightarrow{DTFT} X(e^{j\omega})$$
$$x(n-k) \xleftrightarrow{DTFT} e^{-j\omega k} X(e^{j\omega})$$

$$proof : DTFT\ of\ \ x(n-k)\ is = \sum_{n=-\infty}^{\infty} x(n-k)e^{-j\omega n}$$
$$let\ \ n-k = p\ ; \quad n = k+p$$
$$= \sum_{p=-\infty}^{\infty} x(p)e^{-j\omega(k+p)} = e^{-j\omega k} \sum_{p=-\infty}^{\infty} x(p)e^{-j\omega p} = e^{-j\omega k} X(e^{j\omega})$$

12. State the linearity and periodicity properties of Discrete-Time Fourier Transform.

Linearity:

$$x_1(n) \xleftrightarrow{DTFT} X_1(e^{j\omega});\ x_2(n) \xleftrightarrow{DTFT} X_2(e^{j\omega})$$ where a, b are constants.
$$a\,x_1(n) + b\,x_2(n) \xleftrightarrow{DTFT} a X_1(e^{j\omega}) + bX_2(e^{j\omega})$$

Periodicity:

DTFT is periodic with period 2 π

$$x(n) \xleftrightarrow{DTFT} X(e^{j\omega})$$

$$X(e^{j(\omega+2k\pi)}) = X(e^{j\omega})$$ where „k‟ is an integer.

13. State and prove periodicity property of DTFT. $x(n)$

$$\xleftrightarrow{DTFT} X(e^{j\omega});\ X(e^{j(\omega+2k\pi)}) = X(e^{j\omega})$$

$$proof : \quad X(e^{j(\omega+2\pi)}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j(\omega+2\pi)n} = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}e^{-j2\pi n} = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n} = X(e^{j\omega})$$

$$e^{-j2\pi n} = 1 \quad n = \text{int}\ eger$$

14. If $x[n] \xleftrightarrow{DTFT} X(e^{j\omega})$ **, What is the DTFT of** $x[n] - x[n-1]$ **?**

$$x(n) \xleftrightarrow{DTFT} X(e^{j\omega});\ x(n-1) \xleftrightarrow{DTFT} e^{-j\omega} X(e^{j\omega}) \quad \Rightarrow time\ shifting\ property\ of\ \ DTFT$$

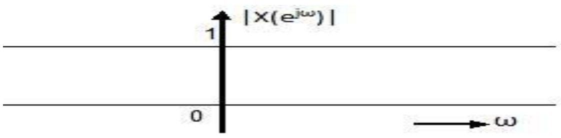
$$DTFT\ of\ \ x[n] - x[n-1]\ is = X(e^{j\omega}) - e^{-j\omega} X(e^{j\omega})$$

15.Compute the discrete time Fourier transform of the signal $x(n) = u(n-2) - u(n-6)$.

$$\xleftrightarrow{DTFT} \frac{1}{\quad} \qquad \xleftrightarrow{DTFT} \frac{e^{-j\omega k}}{\quad}$$ (Time shifting property)

$$\xleftrightarrow{DTFT} \frac{e^{-j2\omega}}{\quad} \qquad 6) \xleftrightarrow{DTFT} \frac{e^{-j\omega 6}}{\quad} \qquad {}_{j\omega}) = \frac{e^{-j2\omega}}{\quad} \quad \frac{e^{-j6\omega}}{\quad}$$

16.Sketch the amplitude spectrum of $\delta[n]$ **;** $x(n) = \delta(n);\ X(e^{j\omega}) = 1$



17. Define unilateral and bilateral Z-transforms.(Dec2013)

$$X(Z) = \sum_{n=0}^{\infty} x(n)z^{-n}$$ -Unilateral ZT ; $$X(Z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$ - Bilateral ZT

18. State the convolution property of the z-transform.(Dec 2012)

$$x_1(n) \xleftrightarrow{z} X_1(z)\ ;\ x_2(n) \xleftrightarrow{z} X_2(z)\ ;\ x_1(n) * x_2(n) \xleftrightarrow{z} X_1(z)\, X_2(z)$$

19. What is aliasing?(May2013)

When sampling rate is less than the Nyquist rate, high frequency fold in and appear as low frequency. The superimposition of the high frequency behaviour on to the low frequency is known as aliasing or frequency folding.

20. State initial value theorem of z – transform.

For causal signal x(n),

$$x(0) = \lim_{z\to\infty} X(z)$$

$$where \quad X(z) = z - transform\ of \quad x(n)$$

21. What is the z transform of δ(n+k)?(May2013)

Using time shifting property of z-transform,

$$X(z) = z^{k}\left[z - transform\ of \quad \delta(n)\right] = z^{k}$$

22. State the final value theorem of z- transform.(May 2012)

For causal signal, $x(n)$; $x(n) \xleftrightarrow{Z+} X^{+}(z)$

If poles of $X^{+}(z)$ are within the unit circle in z-plane, then $x(\infty) = \lim_{z\to 1}(z-1)\,X^{+}(z)$

23.Find the DTFT of x(n)=δ(n)+δ(n-1).(Dec2014)

$X(e^{j\omega})=1+e^{-jw}$

24.State and prove the time folding property of Z-transform. (Dec2014)

Statement: $x(-n)=X(z^{-1})$

Proof:

$$z\{(-n)\} = \sum_{n=-\infty}^{\infty} x(-n)(z^{-n}) = \sum_{n=-\infty}^{\infty} x(-n)(z^{-1})^{-n} = X(z^{-1})$$

25.State the multiplication property of DTFT. (May2014)

Multiplication property:

y[n] = x1[n]x2[n] then

$$Y(e^{j\omega}) = \frac{1}{2\pi}\int_{2\pi} X_1(e^{j\theta})X_2(e^{j(\omega-\theta)})\,d\theta$$

26.For the analog signal(t)=sin(200πt)+3sin^2(120πt), What is the minimum sampling rate required to avoid aliasing? (Apr 15)

$$\sin(200\pi t) \Rightarrow \Omega_1 = 200\pi \quad F_1 = \frac{200\pi}{2\pi} = 100\ Hz$$

$$\sin^2(120\pi t) \Rightarrow \left(\frac{1-\cos(240\pi t)}{2}\right) \quad \Omega_2 = 240\pi F_2 = \frac{240\pi}{2\pi} = 120Hz$$

Highest frequency = 120 Hz = Fm

Minimum sampling rate = Nyquist rate = 2Fm = 240 Hz

27.List the methods to find Inverse Z transform?(Apr15)

1.Partial fraction method , 2.Power series method, 3.Convolution method ,4.Residue method

PART B

1.i)Prove sampling theorem and explain how the original signal can be reconstructed from the sampled version.

ii) State and prove the properties of DTFT.**(Dec 2012)**

2. i) Determine the sequence x(n) from the following function using Partial fraction expansion.

$$X(z) = \frac{1}{1-1.5z^{-1}+0.5z^{-2}} \quad ROC\ is\ |z| > 1$$

ii) Find the DTFT of the signal $x(n) = u(n-2)$

3. i) Determine the Z transform of x(n) = $a^n\cos(\omega_0 n)u(n)$(8) **(May 2013)**

ii) Determine the inverse Z transform of

$$X(z) = X(Z) = \frac{1}{1-1.5z^{-1}+0.5z^{-2}} \quad |Z| > 1\ (8)$$

4. i) State and prove the time shift and frequency shift property of DTFT.(6)

ii) Determine the DTFT of $\left(\frac{1}{2}\right)^n u(n)$.Plotitsspectrum.(10)**(May2013)**

5. i)Determine the Discrete time fourier transform of x(n)= $a^{|n|}$ **,** |a|<1 (8) .**(Dec 2013)**

ii) Find the z-transform and ROC of the sequence x(n)= $r^n\cos(n\theta)u(n)$. (8)

6. i) State and prove the following properties of z-transform. 1) Linearity, 2) Time shifting 3) Differentiation 4)Correlation.(8) **(Dec 2013)**

ii) Find the inverse z-transform of the function (8) $X(z) = \dfrac{1+z^{-1}}{\left(1-\frac{2}{3}z^{-1}\right)} \quad ROC\ is\ |z| > \frac{2}{3}$

7. State and prove sampling theorem for a band limited signal.**(Dec2014)**

8 Find inverse Z-transform of $X(Z) = \dfrac{Z^{-1}}{(1 - 0.25Z^{-1} - 0.375Z^{-2})}$ $|Z| > 0.75$ $|Z| < 0.5$ **(Dec2014)**

9. Using convolution property of DTFT of $X(e^{jw}) = \dfrac{1}{(1-\alpha e^{-jw})^2}$ $|\alpha| < 1$ **(May2014)**

10. Find the inverse Z-transform of $X(Z) = \dfrac{Z^2}{(Z-0.5)(Z-1)^2}$ $|Z| > 1$ (**May2014)**

11. State and explain sampling theorem both in time and frequency domains with necessary quantitative analysis and illustrations. **(Dec2015)**

12. State and prove any two properties of DTFT and any two properties of Z-Transform. **(Dec2015)**

13.i) A continuous time sinusoid $\cos(2\pi ft + \theta)$ is sampled at a rate f_S = 1000Hz. Detremine the resulting signal samples if the input signal frequency f is 400 Hz and 1000 Hz respectively (8) **(May 2015)**

ii) Prove the following DTFT Properties a) nx(n) <-> j d X(Ω)/dΩ b, x(n) $e^{j\Omega_c n}$ <-> $X(\Omega-\Omega_c)$ (8)

14.i)Find the DTFT of $x(n) = (1/2)^{n-1}u(n-1)$ (5)

ii) Using suitable z transform properties find X(z) if $x(n) = (n-2)(1/3)^{n-2}\, u(n-2)$ (6)

iii)Find the z transform of $x(n) = \alpha^{|n|}$ $0 < \alpha < 1$. (5) **(May 2015)**

UNIT V-LINEAR TIME INVARIANT DISCRETE TIME SYSTEMS

PARTA

1. Find the system function for the given difference equation y(n)=0.5y(n-1)+x(n).(May 2012)

Taking Z transform of the equation,

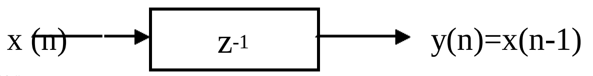
$$Y(z) - 0.5z^{-1}Y(z) = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - 0.5z^{-1}}$$

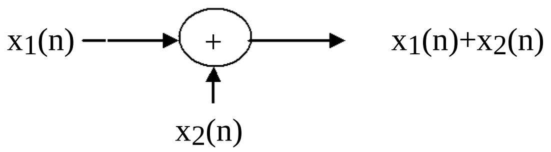
2.What are the basic building blocks to realize any DT system?(Apr 15)

(i) Scalar Multiplier:

(ii) unit delay element:



(iii) summer:



3. What is a F I R system?

FIR-Finite Impulse Response system. Impulse response of the system is of finite duration.

General form of difference equation describing the system is $y(n) = \sum_{k=0}^{M} b_k\, x(n-k)$

4.What is an I I R system?

IIR-infinite impulse response system. Impulse response of the system is of infinite duration. General form of difference equation describing the system is: $y(n) = -\sum_{k=1}^{N} a_k\, y(n-k) + \sum_{k=0}^{M} b_k\, x(n-k)$

5.List the advantages of the state variable representation of a system.(May 2012)

The draw backs of transfer function method of representing system are overcome by state variable representation of a system.Provide information regarding the internal state of a system.

6.Determine the system function of the DT system described by the difference equation

$$y(n) - \frac{1}{2}y(n-1) + \frac{1}{4}y(n-2) = x(n) - x(n-1)$$

Taking Z transform of the equation,

$$Y(z) - \frac{1}{2}z^{-1}Y(z) + \frac{1}{4}z^{-2}Y(z) = X(z) - z^{-1}X(z);\ H(z) = \frac{Y(z)}{X(z)} = \frac{1 - z^{-1}}{1 - \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}} = system \quad function$$

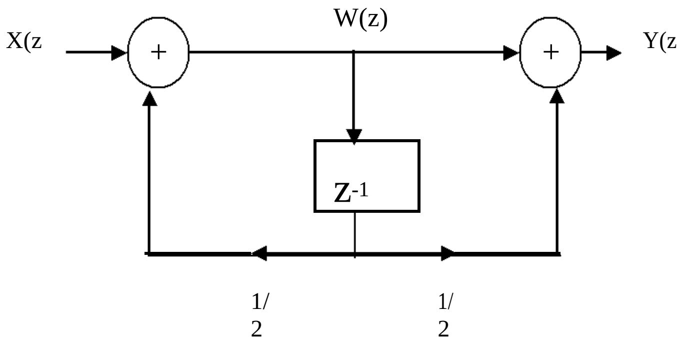
7. Realize the following system using direct form II method.

$y(n) - \frac{1}{2}y(n-1) = x(n) + \frac{1}{2}x(n-1);$ *taking z – transform,*

$$Y(z) - \frac{1}{2}z^{-1}Y(z) = X(z) + \frac{1}{2}z^{-1}X(z) : \frac{Y(z)}{X(z)} = \frac{1+\frac{1}{2}Z^{-1}}{1-\frac{1}{2}z^{-1}} = \frac{Y(z)}{X(z)} \times \frac{W(z)}{W(z)}$$

$$let\ \frac{W(z)}{X(z)} = \frac{1}{1-\frac{1}{2}z^{-1}} \Rightarrow W(z) = X(z) + \frac{1}{2}z^{-1}W(z) \to (1) \frac{Y(z)}{W(z)} = 1 + \frac{1}{2}Z^{-1} \Rightarrow Y(z) = W(z) + \frac{1}{2}z^{-1}W(z) \to (2)$$

Using (1) and (2), the system is realized as follows:



8. Is the discrete time system described by the difference equation y(n) = x(-n) causal? (May 2013)

When n= -1 y(n)=x(-(-1))=x(1) - Future value **;** Therefore the system is non-causal.

9.X(ω) is the DTFT of x(n), what is the DTFT of x^* (-n) ?(May 2013)

$x^*(-n) = X(-\omega)$

10.A Causal LTI system has impulse response $h(n)$, for which the z-transform is

$$H(z) = \frac{1+z^{-1}}{(1-0.5z^{-1})(1+0.25z^{-1})}$$

. Is the system stable? Explain. (Dec 2012)

Poles are at z = 0.5 and at z = -0.25. The poles lie within the unit circle in z-plane. Therefore the system is stable.

11. What are the drawbacks of transfer function representation of the system?

(i)The transfer function describes only the zero state response of a system.(ii) It describes only the relationship between the input and output of a system, but does not provide any information regarding the internal state of a system. (iii) It is limited to single-input single-output systems.(iv) It is applicable only for linear time-invariant systems.

12. In terms of ROC, state the condition for an LTI system discrete time system to be causal and stable. (Dec2014)

A discrete LTI system with rational system function H(z) is causal if and only if theROC is the exterior of the circle of the outer most pole and stable if and only if all of the poles of H(z) lies inside the unit circle.

13.Determine the convolution of the signals x[n]={2,-1,3,2} and h[n]= {1,-1,1,1 (May 2012)

x (n) / h(n)	2	-1	3	2
1	2	-1	3	2
-1	-2	1	-3	-2
1	2	-1	3	2
1	2	-1	3	2

y(n)=x[n]*h[n]={2,-3,6,0,0,5,2}

14. Convolve the following two sequences: x[n]={1,1,1,1} h[n]={3,2}(Dec 2012)

x / h	1	1	1	1
3	3	3	3	3
2	2	2	2	2

y[n]=x[n]*h[n]=[3,5,5,5,2]

15. For a state space representation of the system, find the transfer function of the system.

$A=\begin{pmatrix}0 & 1\\ -3 & -2\end{pmatrix}$, $B=\begin{pmatrix}0\\ 1\end{pmatrix}$, $C=\begin{pmatrix}1 & 2\end{pmatrix}$, $D=(0)$. $H(z)=\dfrac{Y(z)}{X(z)}=C(zI-A)^{-1}B+D$

$A=\begin{pmatrix}0 & 1\\ -3 & -2\end{pmatrix}$, $B=\begin{pmatrix}0\\ 1\end{pmatrix}$, $C=\begin{pmatrix}1 & 2\end{pmatrix}$, $D=(0)$: $zI-A=\begin{pmatrix}z & 0\\ 0 & z\end{pmatrix}-\begin{pmatrix}0 & 1\\ -3 & -2\end{pmatrix}=\begin{pmatrix}z & -1\\ 3 & z+2\end{pmatrix}$

To find $(zI-A)^{-1}$:; $|(zI-A)| = z(z+2)+3$

cofactor of $(zI-A)=\begin{pmatrix}z+2 & -3\\ 1 & z\end{pmatrix}$; $\begin{pmatrix}z+2 & -3\\ 1 & z\end{pmatrix}^T=\begin{pmatrix}z+2 & 1\\ -3 & z\end{pmatrix}$

$$(zI-A)^{-1}=\begin{pmatrix}\dfrac{z+2}{z(z+2)+3} & \dfrac{1}{z(z+2)+3}\\ \dfrac{-3}{z(z+2)+3} & \dfrac{z}{z(z+2)+3}\end{pmatrix};\quad C(zI-A)^{-1}B=\begin{pmatrix}1 & 2\end{pmatrix}\begin{pmatrix}\dfrac{z+2}{z(z+2)+3} & \dfrac{1}{z(z+2)+3}\\ \dfrac{-3}{z(z+2)+3} & \dfrac{z}{z(z+2)+3}\end{pmatrix}\begin{pmatrix}0\\ 1\end{pmatrix}$$

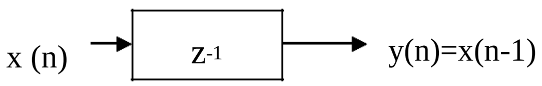
$$=\begin{pmatrix}1 & 2\end{pmatrix}\begin{pmatrix}\dfrac{1}{z(z+2)+3}\\ \dfrac{z}{z(z+2)+3}\end{pmatrix}=\frac{1+2z}{z(z+2)+3}$$

16. State the necessary and sufficient condition for BIBO stability of an LSI system.

For any bounded input signal, the output of the system should be finite(bounded).

17. Write short notes on unit delay element.

It is an important building block of DT system. The output of the element is input delayed by one unit time.



18.Find the transfer function of the system described by the equation y(n-2)-3y(n-1)+2y(n)=x(n-1)

Taking z-transform of the equation,

$$z^{-2}Y(z)-3z^{-1}Y(z)+2Y(z)=z^{-1}X(z);\ H(z)=\frac{Y(z)}{X(z)}=\frac{z^{-1}}{z^{-2}-3z^{-1}+2}$$

19. Check the stability of the system with transfer function $H(z)=\dfrac{1}{\left(1-\frac{1}{2}z^{-1}\right)}$

Pole of the system is 0.5 and it is with in the unit circle in z-plane. Therefore the system is stable.

20.Find the linear convolution of x[n]={1,2,3,4,5,6} with h[n]={2,-4,6,-8}

x(n) / h(n)	1	2	3	4	5	6
2	2	4	6	8	10	12
-4	- 4	-8	-12	-16	-20	-24
6	6	12	18	24	30	36
-8	-8	-16	-24	-32	-40	-48

$x(n) * h(n) = [2,0,4,0,-4,-8,-26,-4,-48]$

21. What is the necessary and sufficient condition for a DT LTI system to be stable?

Necessary and sufficient condition:

Poles of system function H(z) must lie within the unit circle in z - plane.

22. Check whether the system is causal and stable.(Dec 2013)

$$H(Z) = \frac{1}{1-\frac{1}{2}Z^{-1}} + \frac{1}{1-2Z^{-1}} = \frac{(1-2Z^{-1})+(1-\frac{1}{2}Z^{-1})}{(1-1/2Z^{-1})(1-2Z^{-1})}$$

Poles at Z=1/2, Z=2 .The system is causal-Output does not depend on future I/P

The system is unstable since the poles lie outside the unit circle.

23. Given the impulse response of a linear time invariant sytem as h(n)=sinπn, Check whether the system is stable or not.(Nov2014)

Sinπn=0 for n=…-2,-1,0,1,2…

Hence h(n) is absolutely summable and the system is stable.

24.Using Z-Transform check whether the following system is stable.(May2014)

$$H(Z) = \frac{Z}{Z-\frac{1}{2}} + \frac{2Z}{Z-3} \quad ,\frac{1}{2} < |Z| < 3$$

25.Convolve the following signals x(n)=[1,1,3] h(n)=[1,4,-1] (Nov 15)

	1	4	-1
1	1	4	-1
1	1	4	-1
3	3	12	-3

y(n)=[1,5,6,11,-3]

26.Write n^th order Difference equation?(Apr 15)

$$y(n) = -\sum_{k=1}^{N} a_k\, y(n-k) + \sum_{k=0}^{M} b_k\, x(n-k)$$

27.Distinguish Between Recursive and Non Recursive System?(Nov 15)

A recursive system is a system in which present output depends on previous output and input, Non recursive system is a system in which present output depends on previous input.

Recursive

$$y(n) = -\sum_{k=1}^{N} a_k\, y(n-k) + \sum_{k=0}^{M} b_k\, x(n-k)$$

Non Recursive

$$y(n) = \sum_{k=0}^{M} b_k\, x(n-k)$$

IIR filter is example for Recursive system

FIR filter is example for Non Recursive system

PART B

1. i) Find the system function and the impulse response h(n) for a system described by the following input-output relationship. $y(n) = \frac{1}{3} y(n-1) + 3x(n)$

   ii) A linear time-invariant system is characterized by the system function

   $$H(z) = \frac{3-4z^{-1}}{(1-3.5z^{-1}+1.5z^{-2})}.$$

   Specify the ROC of H(z) and determine h(n) for the following conditions:

   1)The system is stable (2)The system is causal (3)The system is anti-causal.**(Dec 2012)**

2. i) Derive the necessary and sufficient condition for BIBO stability of an LSI system.

   ii) Draw the direct form, cascade form and parallel form block diagrams of the following system function:

$$H(z) = \frac{1}{\left(1 + \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{4}z^{-1}\right)}$$ **(Dec 2012)**

3. i) Obtain the impulse response of the system given by the difference equation
   y(n) –(5/6)y(n-1)+(1/6)y(n-2) = x(n)(10)
   ii) Determine the range of values of the parameter "a" for which the LTI system with Impulse response h(n) = a^n u(n) is stable. (6)**(May2013)**
4. Compute the response of the system
   $x(n) = 0.7\,y(n-1) - 0.12y(n-2) + x(n-1) + x(n-2)$ *when input* $x(n) = n\,u(n)$
   (May2013)
5. i)Compute convolution sum of the following sequences (10) **(Dec2013)**
   $x(n) = 1 \quad 0 \le n \le 4$; $=0$ *other wise* ; $h(n) = \alpha^n \quad 0 \le n \le 6$; $=0$ *other wise*
   ii)Draw the direct form –I and direct form-II implementations of the system described by the difference equation (6) $y(n) + \frac{1}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n) + x(n-1).$
6. i) Determine the transfer function and impulse response for the causal LTI system described by the difference equation using z-transform $y(n) - \frac{1}{4}y(n-1) - \frac{3}{8}y(n-2) = -x(n) + 2x(n-1).$
7. Compute y(n) = x(n)* h(n) Where $x(n) = \left(\frac{1}{2}\right)^{-n} u(n-2)$ and $h(n) = u(n-2)$.**(Dec2014)**
8. LTI discrete time system $y(n) = \frac{3}{2}y(n-1) - \frac{1}{2}y(n-2) + x(n) + x(n-1)$ is given an input $x(n) = u(n)$

(i) Find the transfer function of the system.
(ii) Find the impulse response of the system. **(Dec2014)**

9. Find the convolution of sum of x[n] = r[n] and h[n]=u[n].**(May2014)**
10. A casual LTI system is described by $y(n) - \frac{5}{6}y(n-1) + \frac{1}{6}y(n-2) = x(n)$. Find
    (i) System function H(z) (ii) Impulse response h(n).**(May2014)**
11. Convolve the following signals $x(n) = \left(\frac{1}{2}\right)^{n-2} u(n-2)$ *and* $h(n) = u(n+2)$ **(Dec2015)**
12. Consider an LTI system with impulse response h[n] =α^n u[n] and the input to this system is x(n) =β^n u(n) with $|\alpha|$ & $|\beta| < 1$. Determine the response y[n]. i) When α = β and ii) When α ≠ β using DTFT. **(Dec2015)**

13.i) Determine the impulse response and step response of $y(n) + y(n-1) - 2y(n-2) = x(n-1) + 2x(n-2)$
   ii) Find the convolution sum between x(n) =[1,4,3,2] and h(n) = [1,3,2,1] **(May 2015)**

13. i) A causal system has $x(n) = \delta(n) + \frac{1}{4}\delta(n-1) - \frac{1}{8}\delta(n-2)$ $y(n) = \delta(n) - \frac{3}{4}\delta(n-1)$ Find the impulse response and output if $x(n) = \left(\frac{1}{2}\right)^n u(n)$

14. ii) Compare recursive and non- recursive systems **(May 2015)**